



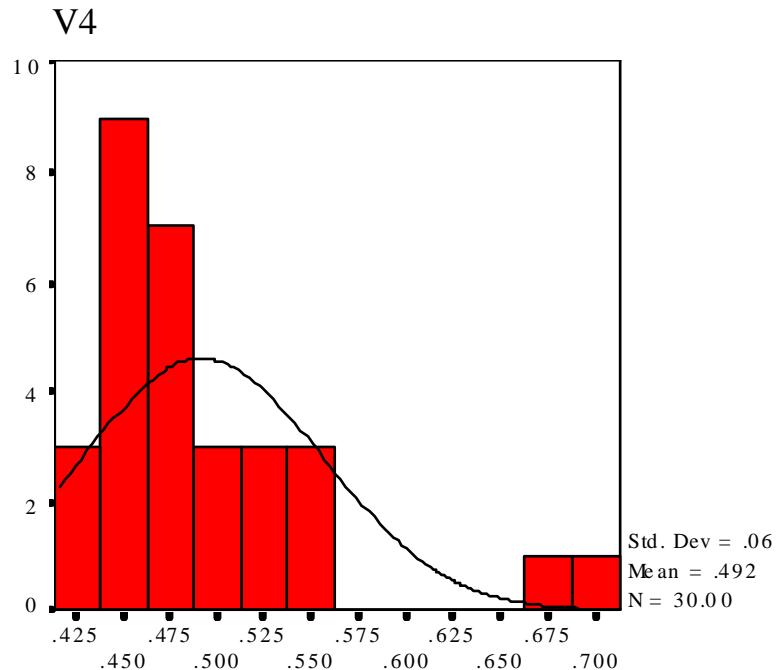
Neparametrični testi

Iztok Grabnar

Univerza v Ljubljani, Fakulteta za farmacijo

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Normalna porazdelitev



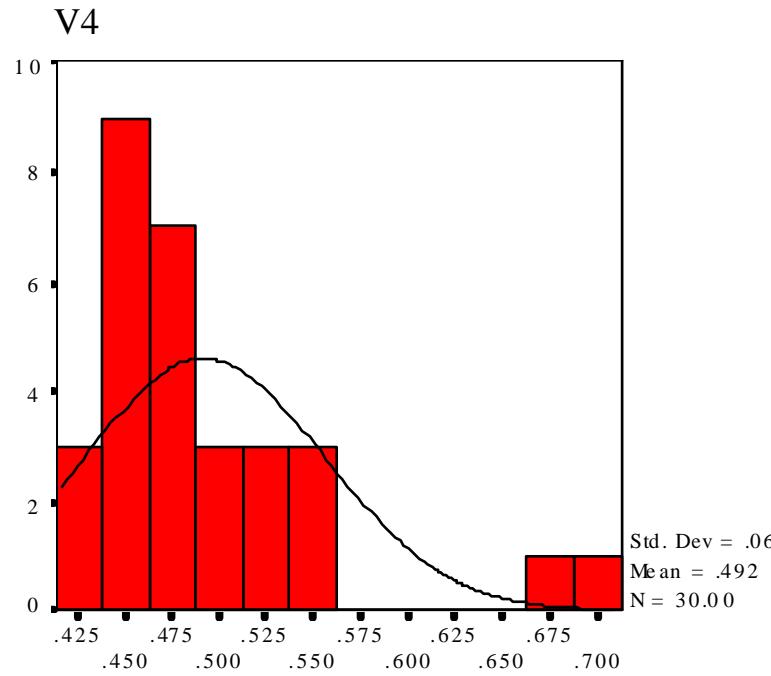
V4

Primerjava aritmetične sredine

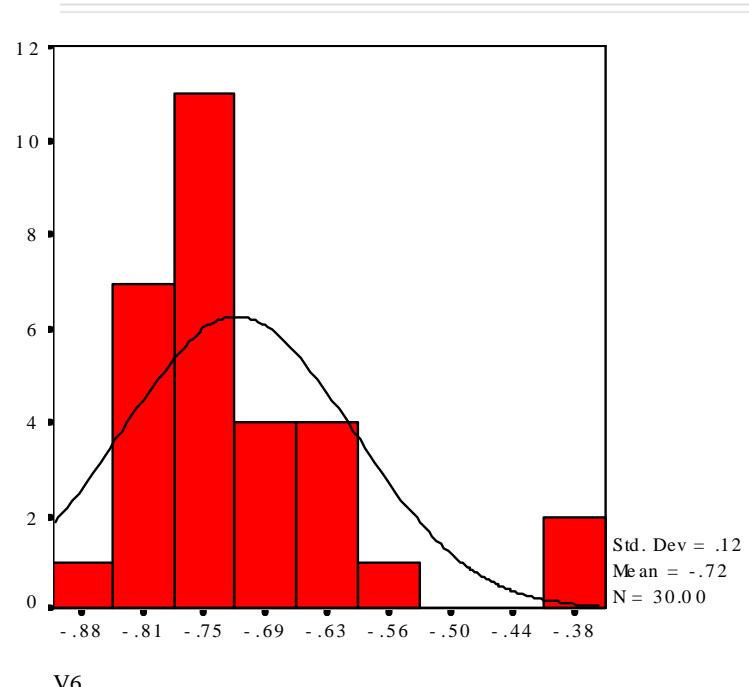
- Vzorčna porazdelitev
 - Centralni limitni izrek
 - Vzorčne statistike
-

Transformacije

- Logaritem (v multiplikativnih modelih)
- Kvadratni koren



V4



Testiranje normalnosti porazdelitve

Testi

- Kolmogorov – Smirnov
 - Lilliefors
 - Shapiro – Wilk
 - χ^2
-

Spremenljivka

- Nominalna
 - Ordinalna
 - Numerična (interval, razmerje)
-

Problem zveznosti in neenakih intervalov

Bolečina

- Brez bolečine (0)
 - Blaga bolečina (1)
 - Zmerna bolečina (2)
 - Huda bolečina (3)
 - Zelo huda bolečina (4)
-

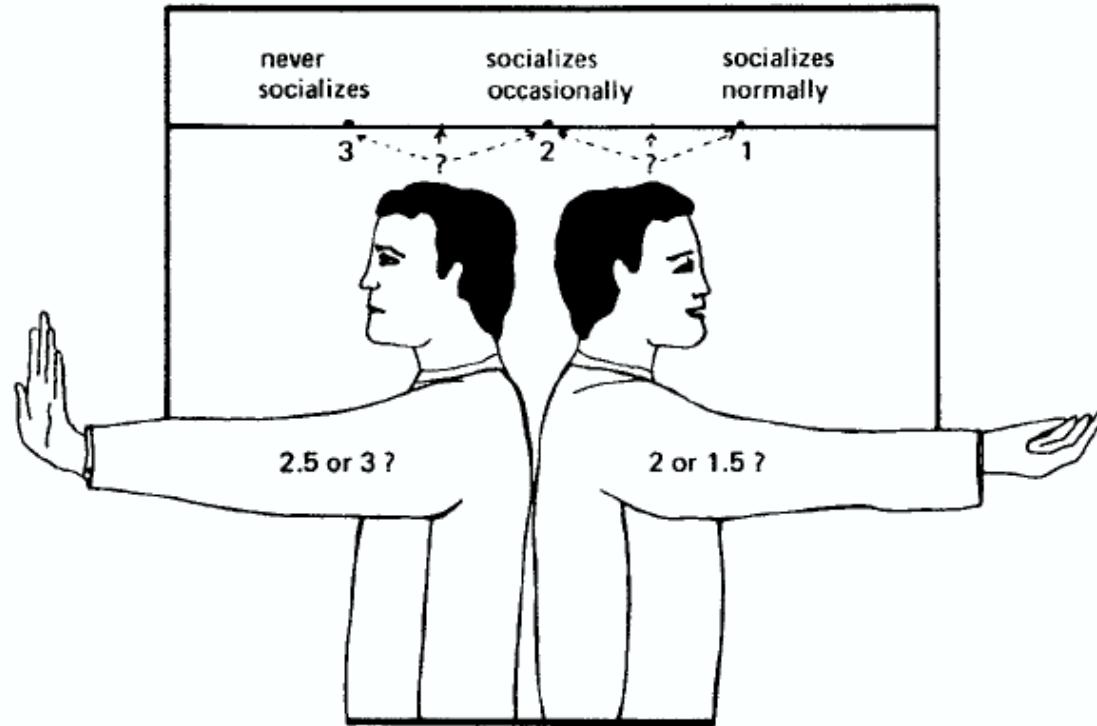


Figure 15.1 Problems with correspondence of a number and a subjective condition.

Merski instrumenti

CGI

CLINICAL GLOBAL IMPRESSIONS

A

1. *Severity of Illness*—Considering your total clinical experience with this particular population, how mentally ill is the patient at this time?

1 = Normal, not at all ill	5 = Markedly ill
2 = Borderline mentally ill	6 = Severely ill
3 = Mildly ill	7 = Among the most extremely ill patients
4 = Moderately ill	

B

2. *Global Improvement*—Rate total improvement whether or not in your judgment it is due entirely to drug treatment. Compared to his condition at admission to the project, how much has he changed?

1 = Very much improved	5 = Minimally worse
2 = Much improved	6 = Much worse
3 = Minimally improved	7 = Very much worse
4 = No change	

VAS

“Visual Analog Scale”

Bolečina

Brez bolečine

Zelo huda bolečina



Neparametrični testi

- Nominalne spremenljivke
 - Ordinalne spremenljivke (osnova je neka spremenljivka z zvezno porazdelitvijo)
 - Odstopanje od normalne porazdelitve
 - V primerjavi s parametričnimi testi je običajno njihova moč manjša!
-

Neparametrični testi

- Nominalne spremenljivke
 - 2 kategoriji (binomska porazdelitev, test za deleže, Fisherjev test)
 - K kategorij (kontingenčne tabele, test χ^2)
 - Ordinalne spremenljivke
 - 2 vzorca
 - Odvisna (test s predznaki, Wilcoxonov test predznačenih rangov)
 - Neodvisna (Wilcoxonov test vsote rangov)
 - K vzorcev (Kruskal-Wallis, Friedman)
-

Level of measurement	One-sample test	Two-sample case		K-sample case	
		Related Samples	Independent samples	Related samples	Independent samples
Nominal	Binomial	McNemar for significance of changes	Fisher exact probability Chi-square	Cochran Q (Dichotomous)	Chi-square
Ordinal	Kolmogorov Smirnov	Sign Wilcoxon matched-pair signed-ranks	Mann-Whitney U Kolmogorov-Smirnov	Friedman two-way analysis of variance	Kruskal-Wallis one-way analysis of variance
	Runs		Wald-Wolfowitz runs Moses of extreme reactions		
Interval		Walsh	Randomization		

Odvisna vzorca

Test s predznaki

Table 15.3 Paired Data Obtained from the Bioavailability Experiment: Time to Peak Plasma Concentration

Subject	Time to peak (hr)		Difference: $B - A$
	A	B	
1	2.5	3.5	+1
2	3.0	4.0	+1
3	1.25	2.5	+1.25
4	1.75	2.0	+0.25
5	3.5	3.5	0
6	2.5	4.0	+1.5
7	1.75	1.5	-0.25
8	2.25	2.5	+0.25
9	3.5	3.0	-0.5
10	2.5	3.0	+0.5
11	2.0	3.5	+1.5
12	3.5	4.0	+0.5

Table IV.12 Number of Positive or Negative Signs Needed for Significance for the Sign Test

Sample size	Number of positive or negative signs for significance ^a	
	5% Level	1% Level
6	6	—
7	7	—
8	8	8
9	8	9
10	9	10
11	10	11
12	10	11
13	11	12
14	12	13
15	12	13
16	13	14
17	13	15
18	14	15
19	15	16
20	15	17

^aThis is a two-sided test. Choose positive or negative signs, whichever is larger.

Test za delež z Yatesovo korekturo

$N > 20$

$$Z = \frac{|P - 0.5| - 1/(2N)}{0.5 / \sqrt{N}}$$

Wilcoxonov test predznačenih rangov

Porazdelitvi sta simetrični

Table 15.3 Paired Data Obtained from the Bioavailability Experiment: Time to Peak Plasma Concentration

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1	2.5	3.5	+1
2	3.0	4.0	+1
3	1.25	2.5	+1.25
4	1.75	2.0	+0.25
5	3.5	3.5	0
6	2.5	4.0	+1.5
7	1.75	1.5	-0.25
8	2.25	2.5	+0.25
9	3.5	3.0	-0.5
10	2.5	3.0	+0.5
11	2.0	3.5	+1.5
12	3.5	4.0	+0.5

Table 15.4 Data from Table 15.3: Ranking Differences Without Regard to Sign for the Wilcoxon Signed Rank Test

Subject	Value	Rank	Assigned rank	Assigned rank with sign
7	-0.25	1	2	-2
4	0.25	2	2	2
8	0.25	3	2	2
9	-0.5	4	5	-5
10	0.5	5	5	5
12	0.5	6	5	5
1	1.0	7	7.5	7.5
2	1.0	8	7.5	7.5
3	1.25	9	9	9
6	1.5	10	10.5	10.5
11	1.5	11	10.5	10.5
Ranks with positive signs			Ranks with negative signs	
2			2	
2			<u>5</u>	
5			sum = 7	
5				
7.5				
7.5				
9				
10.5				
<u>10.5</u>				
sum = 59				

Table IV.13 Values Leading to Significance for the Wilcoxon Signed Rank Test (Two-Sided Test)

Sample size, N	5% Level ^a	1% Level
6	0	—
7	2	—
8	3	0
9	5	1
10	8	3
11	10	5
12	13	7
13	17	10
14	21	13
15	25	16
16	30	19
17	35	23
18	40	28
19	46	32
20	52	37

^aIf the smaller rank sum is less than or equal to the table value, the comparative groups are different at the indicated level of significance.

Normalna aproksimacija

$$Z = \frac{|R - N(N+1)/4|}{\sqrt{[N(N+1/2)(N+1)]/12}}$$

$$Z = \frac{|59 - 11(12)/4|}{\sqrt{11(11.5)(12)/12}} = 2.31$$

Neodvisna vzorca Wilcoxonov test vsote rangov Mann-Whitney U-test

Table 15.8 Results of a Dissolution Test Using the Original Dissolution Apparatus and a Modification: Amount Dissolved in 30 Minutes

Original apparatus		Modified apparatus	
Amount dissolved	Rank	Amount dissolved	Rank
53	3	58	11
61	14	55	5.5
57	9	67	21
50	1	62	15.5
63	17	55	5.5
62	15.5	64	18.5
54	4	66	20
52	2	59	12.5
59	12.5	68	22
57	9	57	9
64	18.5	69	23
		56	7
Sum of ranks	105.5		170.5

Table IV.16 Critical Values for Wilcoxon Rank Sum Test^a ($\alpha = 0.05$)

Size of larger sample	Size of smaller sample (M)						
	$M = 3$	4	5	6	7	8	9
M	5, 16	11, 25	18, 37	26, 52	37, 68	49, 87	63, 108
$M + 1$	6, 18	12, 28	19, 41	28, 56	39, 73	51, 93	66, 114
$M + 2$	6, 21	12, 32	20, 45	29, 61	41, 78	54, 98	68, 121
$M + 3$	7, 23	13, 35	21, 49	31, 65	43, 83	56, 104	71, 127
$M + 4$	7, 26	14, 38	22, 53	32, 70	45, 88	58, 110	74, 133
$M + 5$	8, 28	15, 41	24, 56	34, 74	46, 94	61, 115	77, 139
$M + 6$	8, 31	16, 44	25, 60	36, 78	48, 99	63, 121	79, 146
$M + 7$	9, 33	17, 47	26, 64	37, 83	50, 104	65, 127	82, 152
$M + 8$	10, 35	17, 51	27, 68	39, 87	52, 109	68, 132	85, 158
$M + 9$	10, 38	18, 54	29, 71	41, 91	54, 114	70, 138	88, 164
$M + 10$	11, 40	19, 57	30, 75	42, 96	56, 119	72, 144	90, 171
$M + 15$	13, 53	24, 72	36, 94	50, 118	66, 144	84, 172	104, 202
$M + 20$	16, 65	28, 88	42, 113	58, 140	76, 169	96, 200	118, 223
$M + 25$	18, 78	32, 104	48, 132	66, 162	86, 194	108, 228	132, 264

^aFrom Wilcoxon, F. and Wilcox, R. A., *Some Rapid Approximate Statistical Procedures*, Lederle Laboratories, 1964.

If rank sum of smaller sample is equal to or lower than smaller numbers in table or equal to or larger than larger number, groups are significantly different at 0.05 level.

Normalna aproksimacija

$$Z = \frac{|T - N_1(N_1 + N_2 + 1)/2|}{\sqrt{N_1 N_2 (N_1 + N_2 + 1)/12}}$$

$$Z = \frac{|105.5 - 11(11+12+1)/2|}{\sqrt{(11)(12)(11+12+1)/12}} = \frac{26.5}{16.25} = 1.63$$

$$t = \frac{61.3 - 57.45}{5.05 \sqrt{1/12 + 1/11}} = 1.84$$

Kruskal-Wallis

Table 15.10 “Time to Sleep” for a Control and Two Doses of an Experimental Compound (min)

Control	Rank	Low dose	Rank	High dose	Rank
8	22	10	26	3	10
1	3.5	5	13	4	12
9	24.5	8	22	8	22
		6	15	1	3.5
9	24.5	7	18.5	1	3.5
6	15	7	18.5	3	10
3	10	15	28	1	3.5
15	28	1	3.5	6	15
1	3.5	15	28	2	7.5
7	<u>18.5</u>	7	<u>18.5</u>	2	<u>7.5</u>
Sum of ranks	149.5		191.0		94.5

Table IV.17 Critical Difference for Significance ($\alpha = 0.05$) Comparing All Possible Pairs of Treatments for Nonparametric One-Way ANOVA^a

<i>N</i> (for each treatment)	Number of treatments				
	3	4	5	6	7
3	15	23	30	37	45
4	24	35	46	57	69
5	33	48	63	79	96
6	43	63	83	104	125
7	54	79	105	131	158
8	66	96	128	160	192
9	79	115	152	190	229
10	92	134	178	223	268
11	106	155	205	257	309
12	121	176	233	292	352
13	136	199	263	329	397
14	152	222	294	368	444
15	169	246	326	408	492
16	186	271	359	449	542
17	203	296	393	492	593
18	221	323	428	536	646
19	240	350	464	581	700
20	259	378	501	627	756
21	278	406	538	674	814
22	298	435	577	723	872
23	319	465	617	773	932
24	340	496	657	824	994
25	361	527	699	875	1056

^aFrom Wilcoxon, F. and Wilcox, R. A., *Some Rapid Approximate Statistical Procedures*, Lederle Laboratories, 1964.

Kruskal-wallis statistika

$$\chi^2_{k-1} = \frac{12}{N(N+1)} \left(\sum \frac{R_i^2}{n_i} \right) - 3(N+1)$$

$$\chi^2_2 = \frac{12}{(29)(30)} \left(\frac{149.5^2}{9} + \frac{191^2}{10} + \frac{94.5^2}{10} \right) - 3(29+1) = 6.89$$

Korekcija

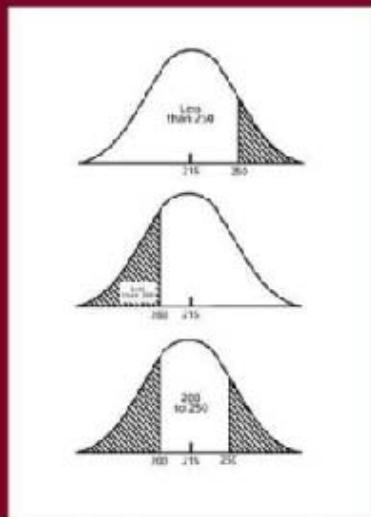
$$\text{Correction} = \frac{\chi^2}{1 - \sum(t_i^3 - t_i)/(N^3 - N)}$$

$$\frac{6.89}{1 - 378/(29^3 - 29)} = \frac{6.89}{0.984} = 7.00$$

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Poglavlje 15!