



Testiranje hipotez o razliki med dvema vzorcema

Iztok Grabnar

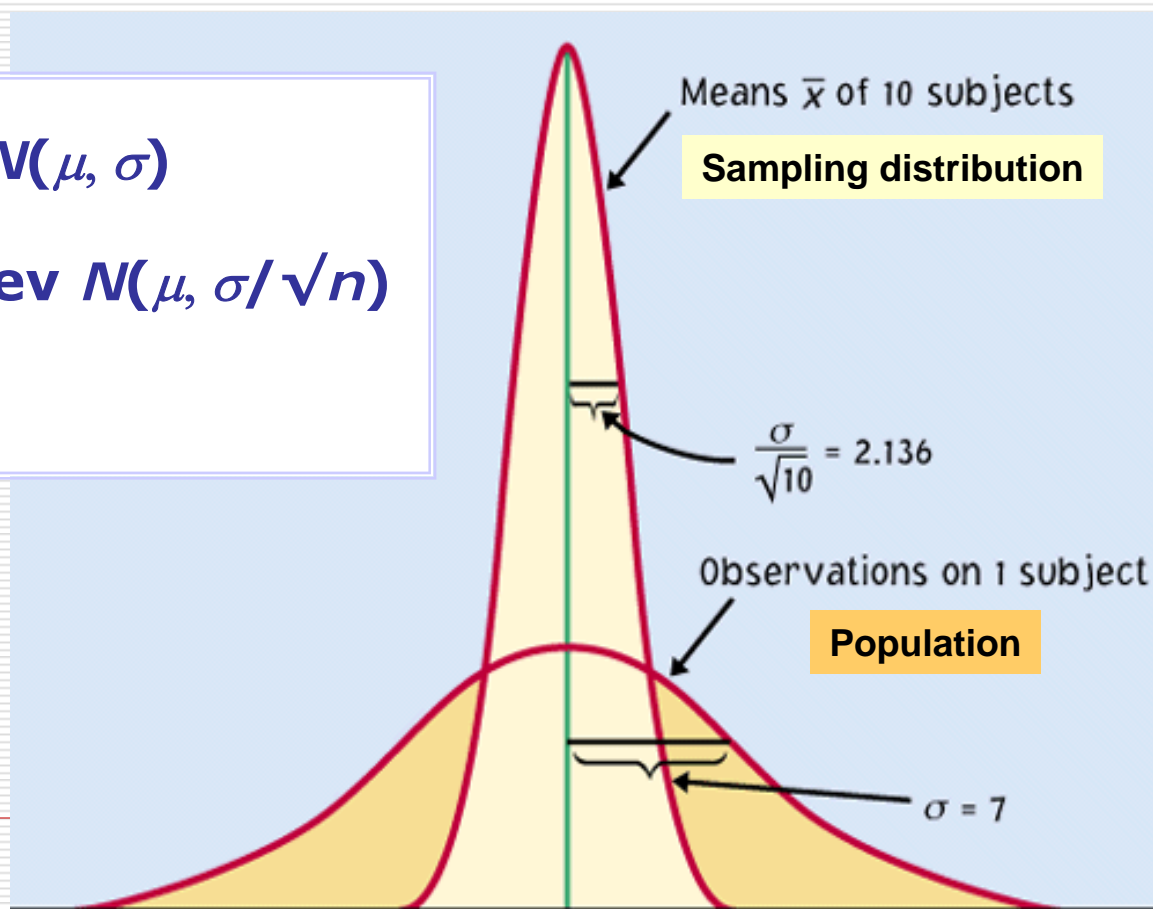
Univerza v Ljubljani, Fakulteta za farmacijo

April, 2009

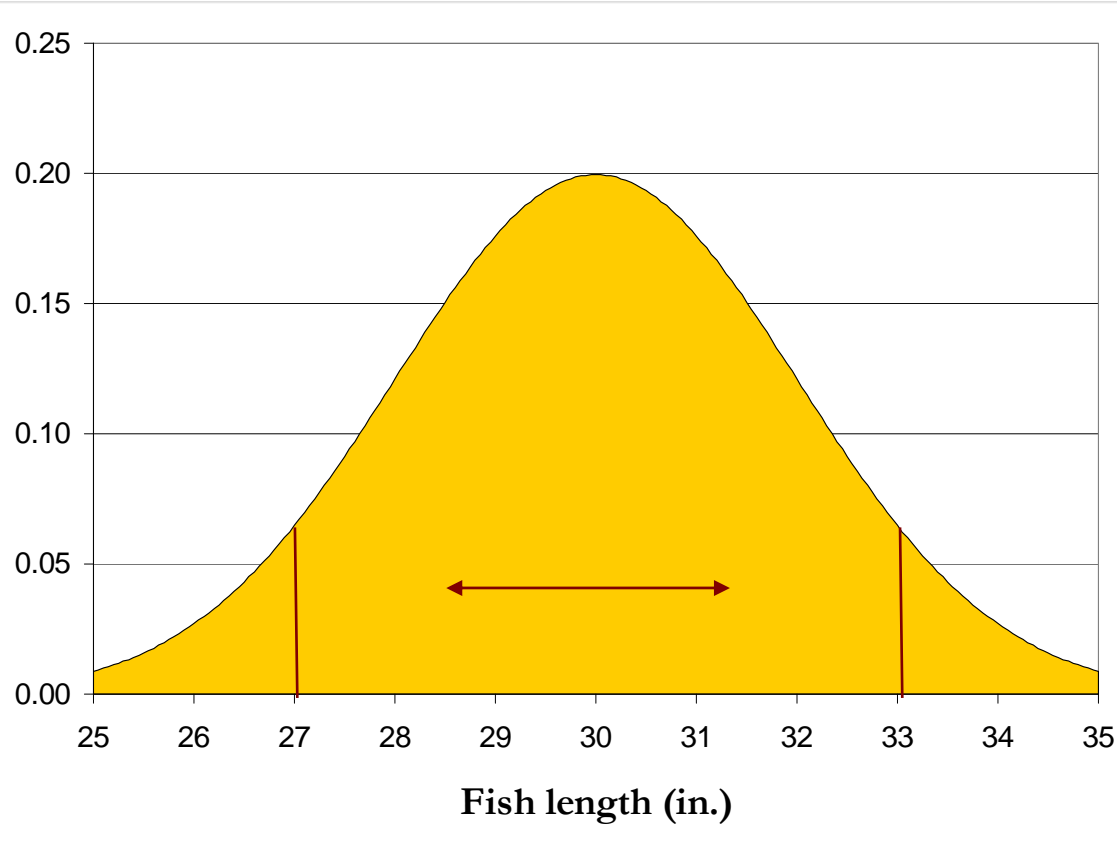
Centralni limitni izrek

Populacija $N(\mu, \sigma)$

Vzorčna porazdelitev $N(\mu, \sigma/\sqrt{n})$



Intervalna ocena za aritmetično sredino



Poznana varianca

$$\bar{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{N}} \leq \mu \leq \bar{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{N}}$$

| Stopnja tveganja (α) | Enostransko tveganje | Dvostransko tveganje |
|-------------------------------|----------------------|----------------------|
| 0.1 | 1.282 | 1.645 |
| 0.05 | 1.645 | 1.960 |
| 0.025 | 1.960 | 2.241 |
| 0.01 | 2.326 | 2.576 |
| 0.005 | 2.576 | 2.807 |

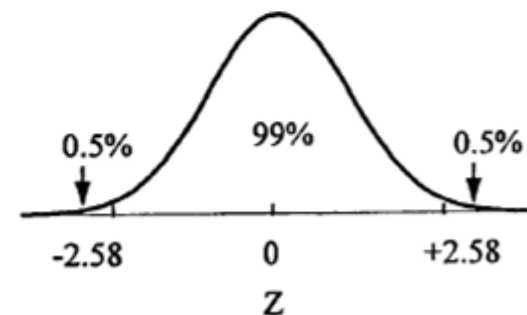
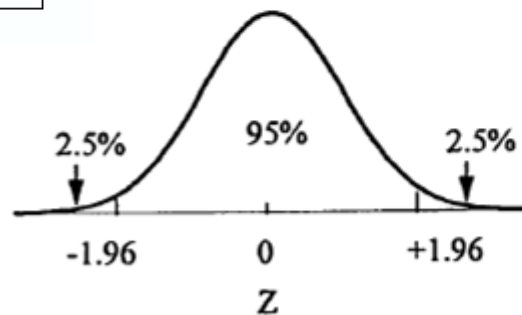
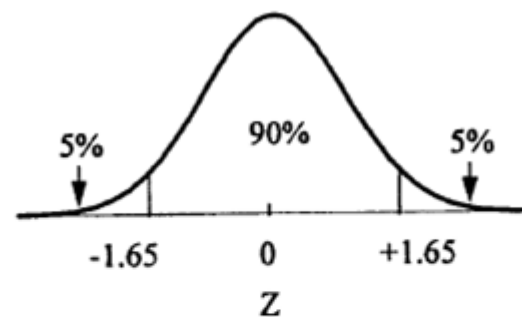
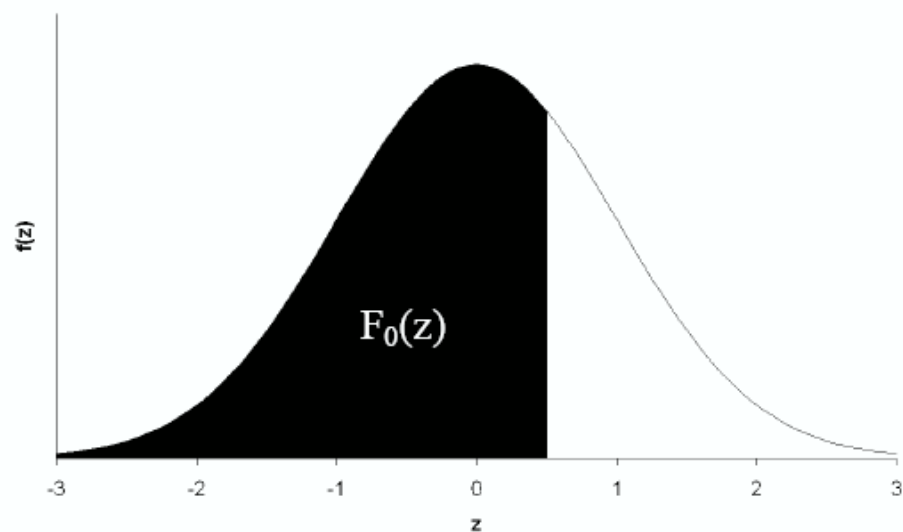


Tabela standardizirane normalne distribucije.



| Stopnja tveganja (α) | Enostransko tveganje | Dvostransko tveganje |
|-------------------------------|----------------------|----------------------|
| 0.1 | 1.282 | 1.645 |
| 0.05 | 1.645 | 1.960 |
| 0.025 | 1.960 | 2.241 |
| 0.01 | 2.326 | 2.576 |
| 0.005 | 2.576 | 2.807 |

$F_0(z)$

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |

Prava varianca ni poznana Ocenjena iz vzorca

$$\bar{X} - t_{\alpha/2} \frac{s}{\sqrt{N}} \leq \mu \leq \bar{X} + t_{\alpha/2} \frac{s}{\sqrt{N}}$$

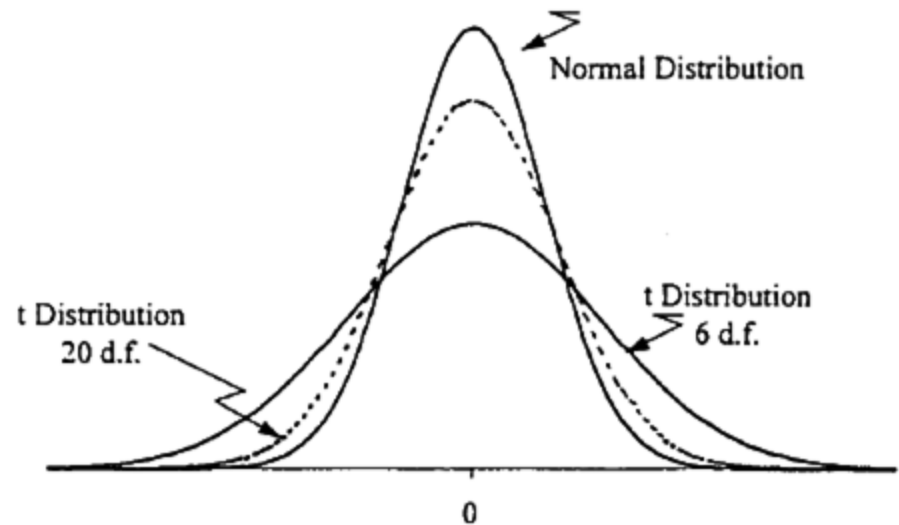
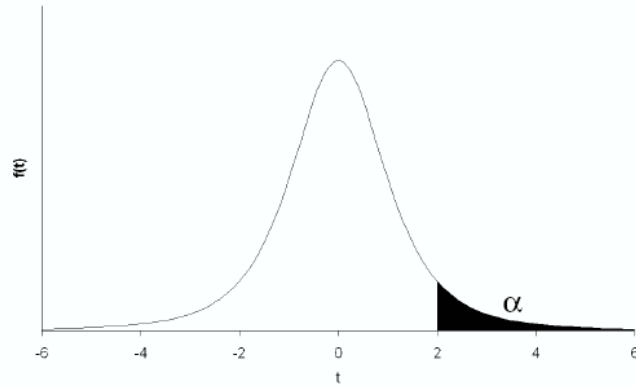


Figure 5.9 *t* distribution compared to the standard normal distribution.

Studentova porazdelitev



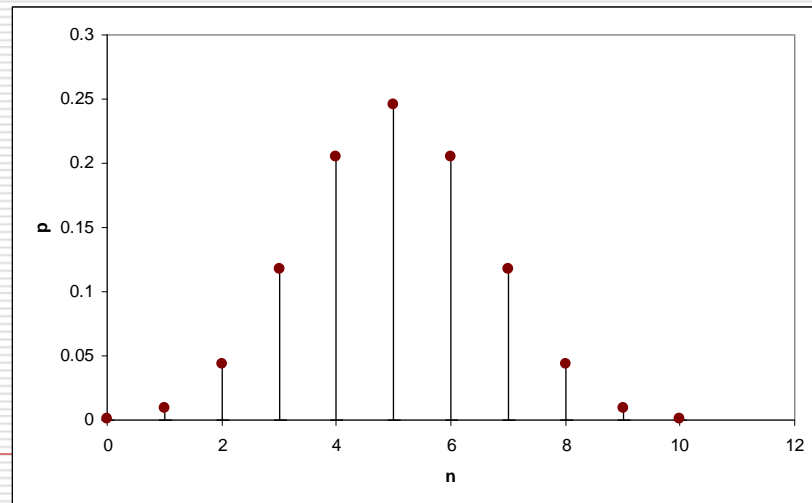
t

| Število prostostnih stopenj | Stopnja tveganja (α) | | | | |
|-----------------------------|-------------------------------|-------|-------|--------|--------|
| | 0.4 | 0.2 | 0.1 | 0.05 | 0.01 |
| Dvostransko tveganje | | | | | |
| Enostransko tveganje | | | | | |
| 1 | 1.376 | 3.078 | 6.314 | 12.706 | 63.656 |
| 2 | 1.061 | 1.886 | 2.920 | 4.303 | 9.925 |
| 3 | 0.978 | 1.638 | 2.353 | 3.182 | 5.841 |
| 4 | 0.941 | 1.533 | 2.132 | 2.776 | 4.604 |
| 5 | 0.920 | 1.476 | 2.015 | 2.571 | 4.032 |
| 6 | 0.906 | 1.440 | 1.943 | 2.447 | 3.707 |
| 7 | 0.896 | 1.415 | 1.895 | 2.365 | 3.499 |
| 8 | 0.889 | 1.397 | 1.860 | 2.306 | 3.355 |
| 9 | 0.883 | 1.383 | 1.833 | 2.262 | 3.250 |

Intervalna ocena za delež

Aproksimacija z normalno porazdelitvijo

$$\hat{p} - Z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{N}} \leq p \leq \hat{p} + Z_{\alpha} \sqrt{\frac{\hat{p}\hat{q}}{N}}$$



Preizkušanje statističnih hipotez

- H_0 : ničelna hipoteza
- H_a : alternativna hipoteza

Table 5.6 Alpha and Beta Probabilities in Hypothesis Testing (Errors When Accepting or Rejecting H_0)

| | H_0 is true | H_a (a specific alternative) is true |
|-------------------|--------------------|--|
| H_0 is rejected | Alpha (α) | 1 – beta |
| H_0 is accepted | 1 – alpha | Beta (β) |

Absence of evidence of the effect is not evidence of absence of the effect

Preizkušanje statističnih hipotez za en vzorec

Table 5.5 Results of 20 Single-Tablet Assays from a Modification of a Process with a Historical Mean of 5.01 mg

| | | | |
|------|------|------|------|
| 5.13 | 5.04 | 5.09 | 5.00 |
| 4.98 | 5.03 | 5.01 | 4.99 |
| 5.20 | 5.08 | 4.96 | 5.18 |
| 5.08 | 5.06 | 5.02 | 5.24 |
| 4.99 | 5.17 | 5.06 | 5.00 |

$$\bar{X} = 5.0655 \text{ mg} \quad S = 0.0806$$

$$\sigma (\text{historical}) = 0.11$$

En vzorec, prava varianca je poznana

Enostransko tveganje

Enostranski test

$$H_0: \mu \geq b$$

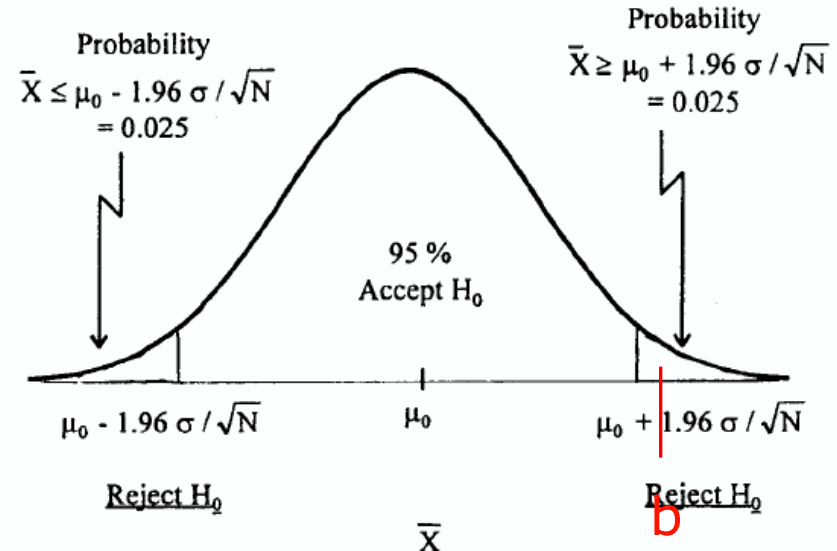
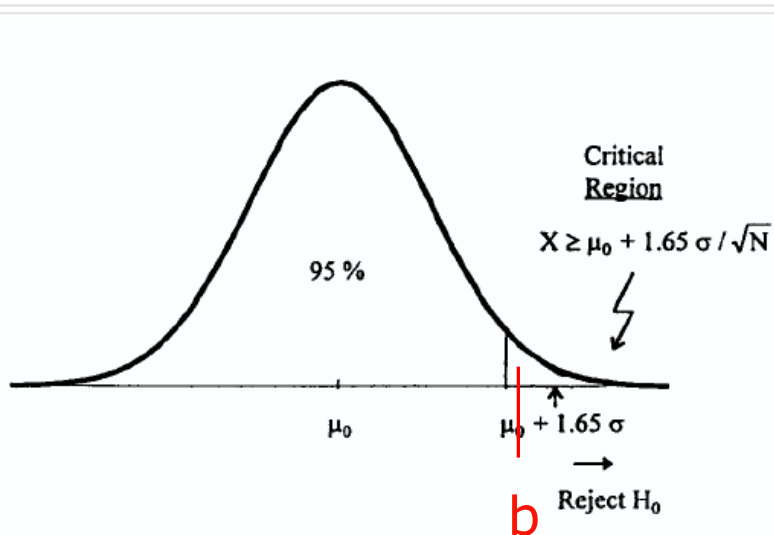
$$H_a: \mu < b$$

Dvostransko tveganje

Dvostranski test

$$H_0: \mu = b$$

$$H_a: \mu \neq b$$



En vzorec, prava varianca je poznana

Enostransko tveganje

$$H_0: \mu - b \geq 0$$

$$H_a: \mu - b < 0$$

$$Z_{\text{exp}} = \frac{\mu - b}{\frac{\sigma}{\sqrt{N}}}$$

$$Z_{\text{tab}} = -1.645$$

Dvostransko tveganje

Dvostranski test

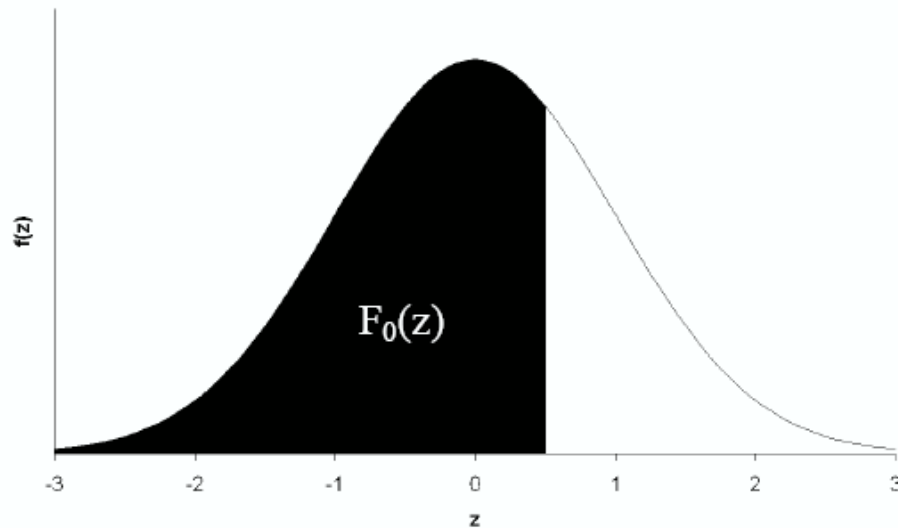
$$H_0: \mu = b$$

$$H_a: \mu \neq b$$

$$Z_{\text{exp}} = \frac{|\mu - b|}{\frac{\sigma}{\sqrt{N}}}$$

$$Z_{\text{tab}} = 1.96$$

$$\alpha = 0.05$$



$$Z_{\text{exp}} > Z_{\text{tab}} \Leftrightarrow 1 - F_0(Z_{\text{exp}}) < \alpha$$

H_0 zavrnemo in sprejmemo H_a
 Razlika je statistično značilna!

Enostransko tveganje

~~$$H_0: \mu - b \geq 0$$

$$H_a: \mu - b < b$$~~

Dvostransko tveganje

Dvostranski test

~~$$H_0: \mu - b = 0$$

$$H_a: \mu - b \neq 0$$~~

En vzorec, prava varianca ni poznana t-test za en vzorec

Enostransko tveganje

$$H_0: \mu - b \geq 0$$

$$H_a: \mu - b < 0$$

Dvostransko tveganje

Dvostranski test

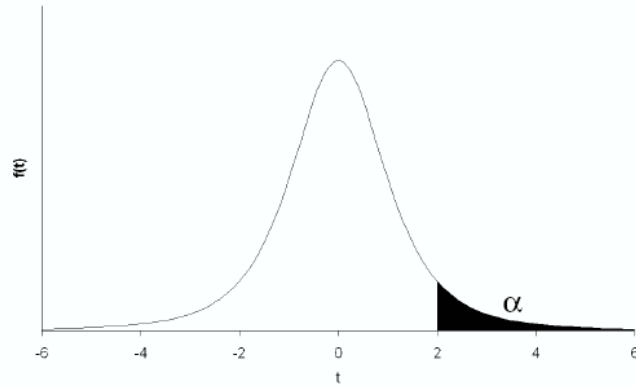
$$H_0: \mu = b$$

$$H_a: \mu \neq b$$

$$t_{\text{exp}} = \frac{\mu - b}{\frac{s}{\sqrt{N}}}$$

$$t_{\text{exp}} = \frac{|\mu - b|}{\frac{s}{\sqrt{N}}}$$

Studentova porazdelitev



t

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| 8 | 0.889 | 1.397 | 1.860 | 2.306 | 3.355 |
| 9 | 0.883 | 1.383 | 1.833 | 2.262 | 3.250 |

Dva neodvisna vzorca t-test za dva neodvisna vzorca

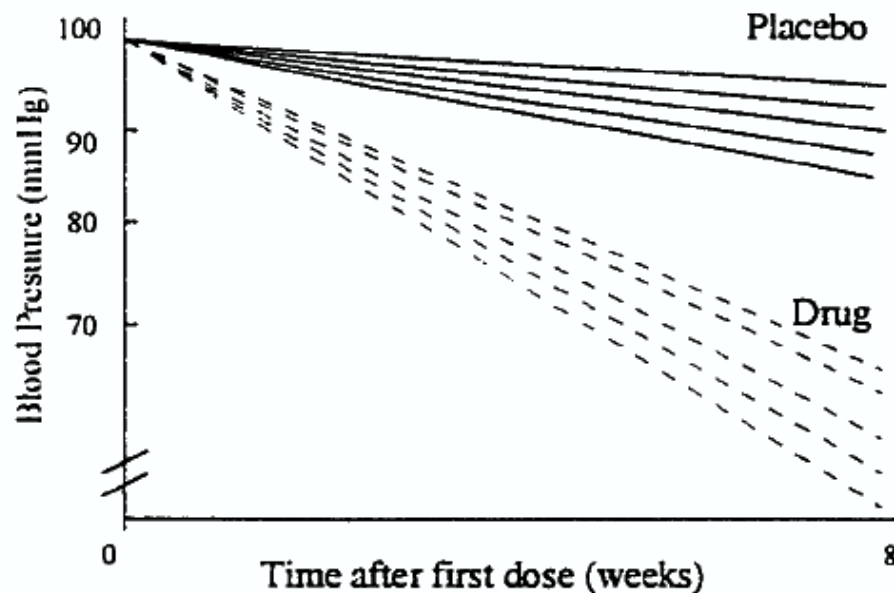
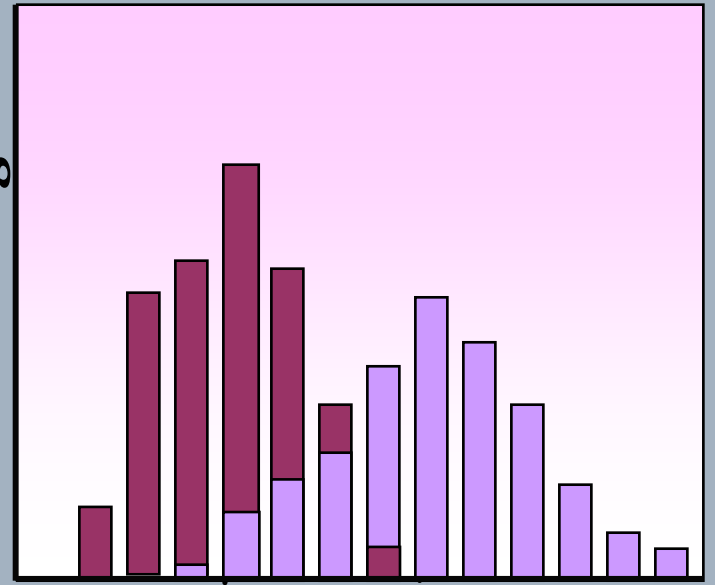


Figure 5.6 Mark of a real drug effect: A large difference between drug and placebo with small variation.

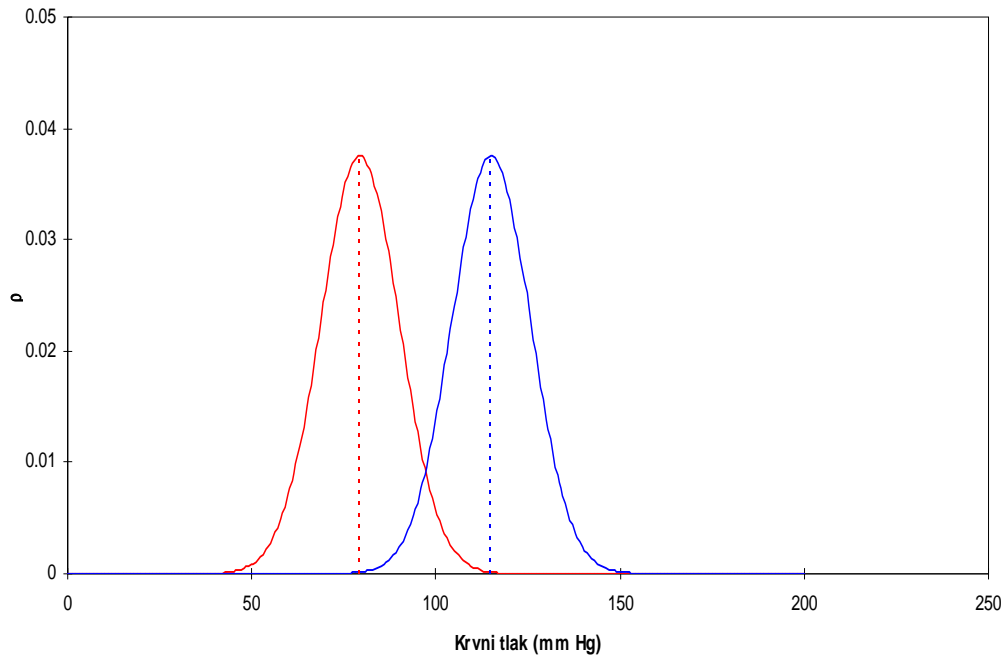


**Blood pressure
mm of Hg**



Groups

-  **Control**
-  **Treatment**



Dva vzorca s poznanimi variancama

$$Z_{\text{exp}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$

Pravi varianci nista poznani, a sta enaki Vzorčna ocena skupne variance

$$s_p^2 = \frac{\sum (x_{1,i} - \bar{X}_1)^2 + \sum (x_{2,i} - \bar{X}_2)^2}{N_1 - 1 + N_2 - 1}$$

$$s_p^2 = \frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{N_1 + N_2 - 2}$$

$$10 < N_1 \text{ in } N_2 < 20 \quad s_1^2 / s_2^2 < 4$$

$$t_{\text{exp}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{N_1} + \frac{s_p^2}{N_2}}}$$

$$t_{\text{tab}}(\alpha, df = N_1 + N_2 - 2)$$

Pravi varianci nista poznani, a sta različni Behrens-Fisher test

$$t'_{\text{exp}} = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

t_{tab}

$$N_1 = N_2 = N$$
$$\text{df} = N - 1$$

$$N_1 \neq N_2$$
$$t_{\text{tab}} = (w_1 t_1 + w_2 t_2) / (w_1 + w_2)$$
$$w_1 = s_1^2 / N_1, w_2 = s_2^2 / N_2$$

Test enakosti varianc

F-test

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2$$

$$F_{\text{exp}} = \frac{s_1^2}{s_2^2}$$

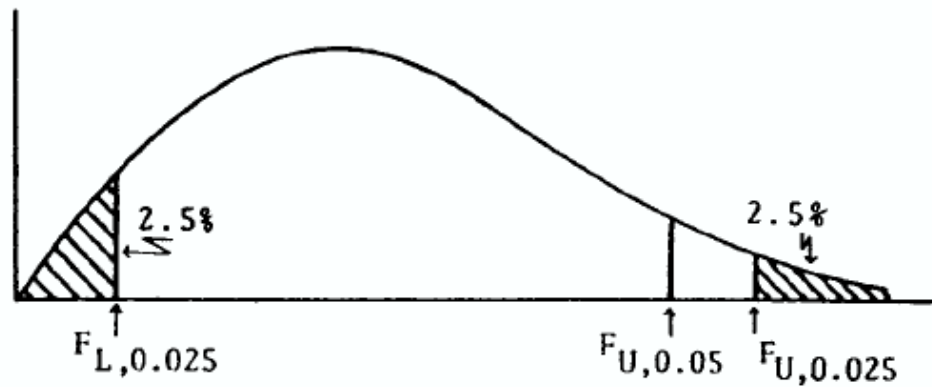


Figure 5.12 Example of two-sided cutoff points in an F distribution.

Dva odvisna vzorca parni t-test

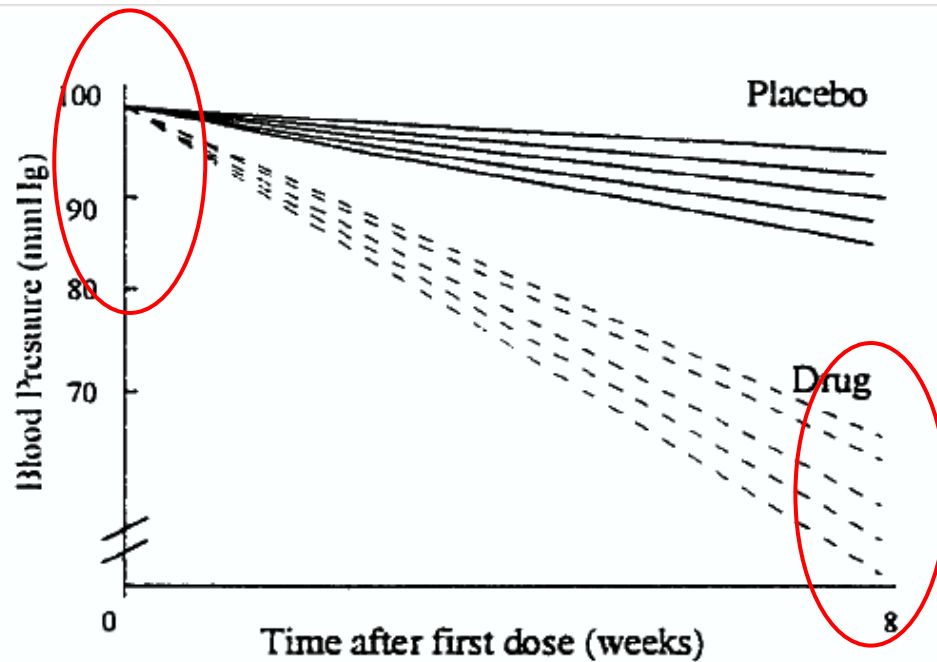


Figure 5.6 Mark of a real drug effect: A large difference between drug and placebo with small variation.

Deleži

En vzorec

$$Z_{\text{exp}} = \frac{|\hat{p} - p_0| + \frac{1}{2N}}{\sqrt{\frac{p_0 q_0}{N}}}$$

Yatesova korektura

Dva neodvisna vzorca

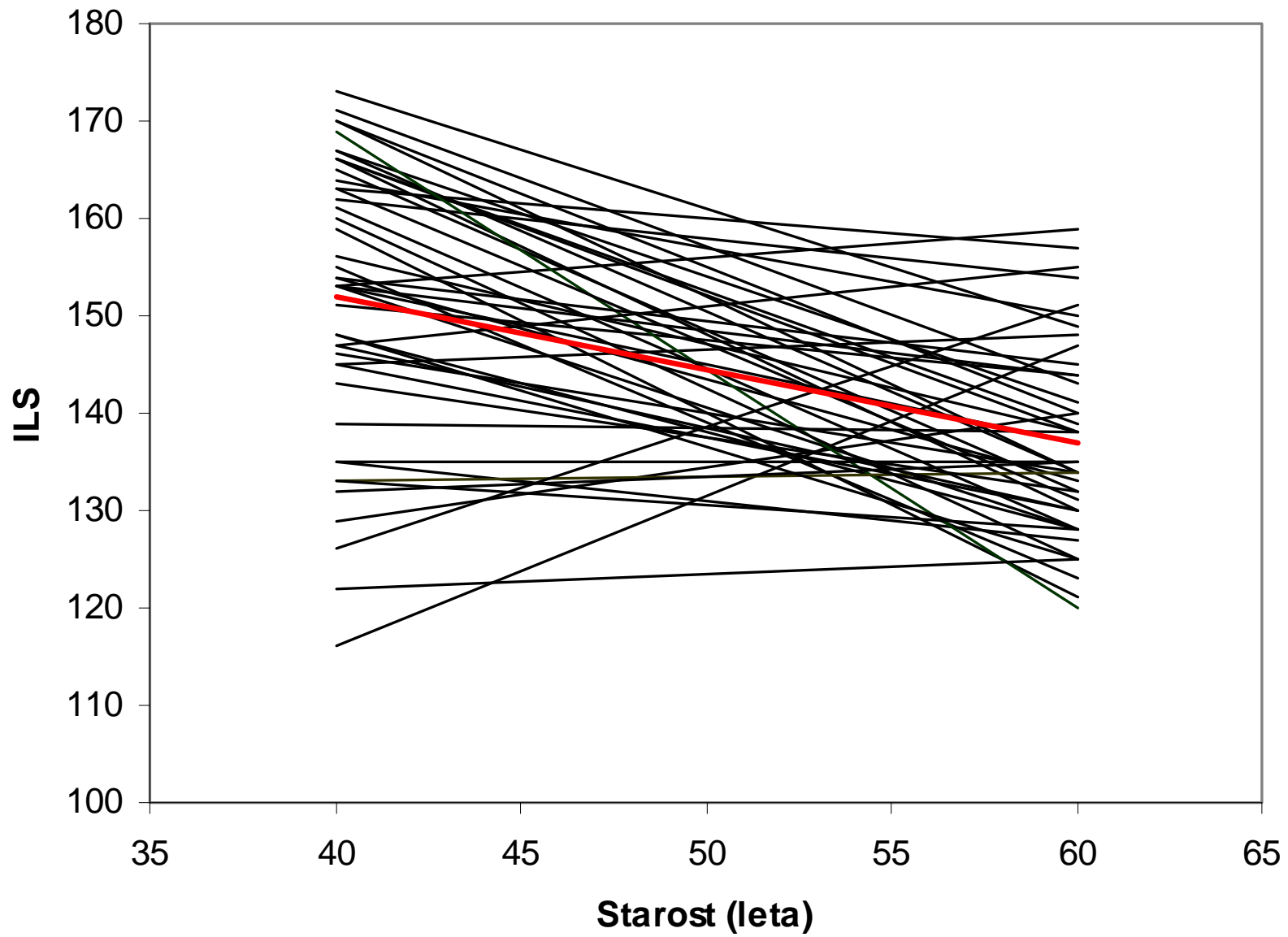
$$Z_{\text{exp}} = \frac{|\hat{p}_a - \hat{p}_b|}{\sqrt{\frac{p_0 q_0}{N_a} + \frac{p_0 q_0}{N_b}}}$$

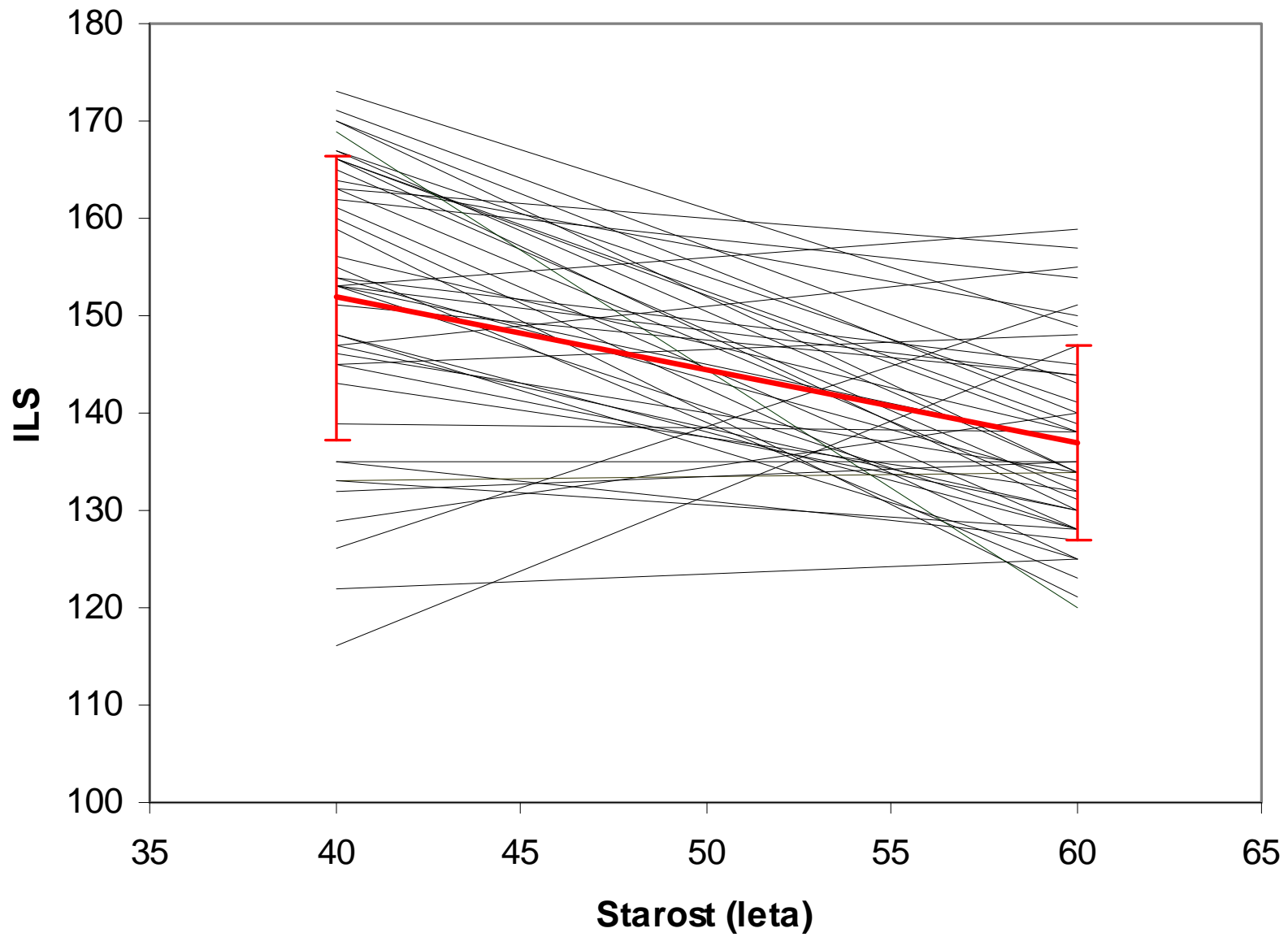
$$p_0 = \frac{\hat{p}_a N_a + \hat{p}_b N_b}{N_a + N_b}$$

Primer 1

Ali se stres s starostjo spremeni?

ILS (Index of Life Stress) pri skupini 100 žensk starosti 40 let. Drugi vzorec pridobljen pri podskupini 45 žensk, ko so stare 60 let.





T-test za dva odvisna vzorca

Paired Samples Statistics

| | | Mean | N | Std. Deviation | Std. Error Mean |
|------|--------|--------|----|----------------|-----------------|
| Pair | ILS 40 | 151.84 | 45 | 14.483 | 2.159 |
| 1 | ILS 60 | 136.87 | 45 | 9.949 | 1.483 |

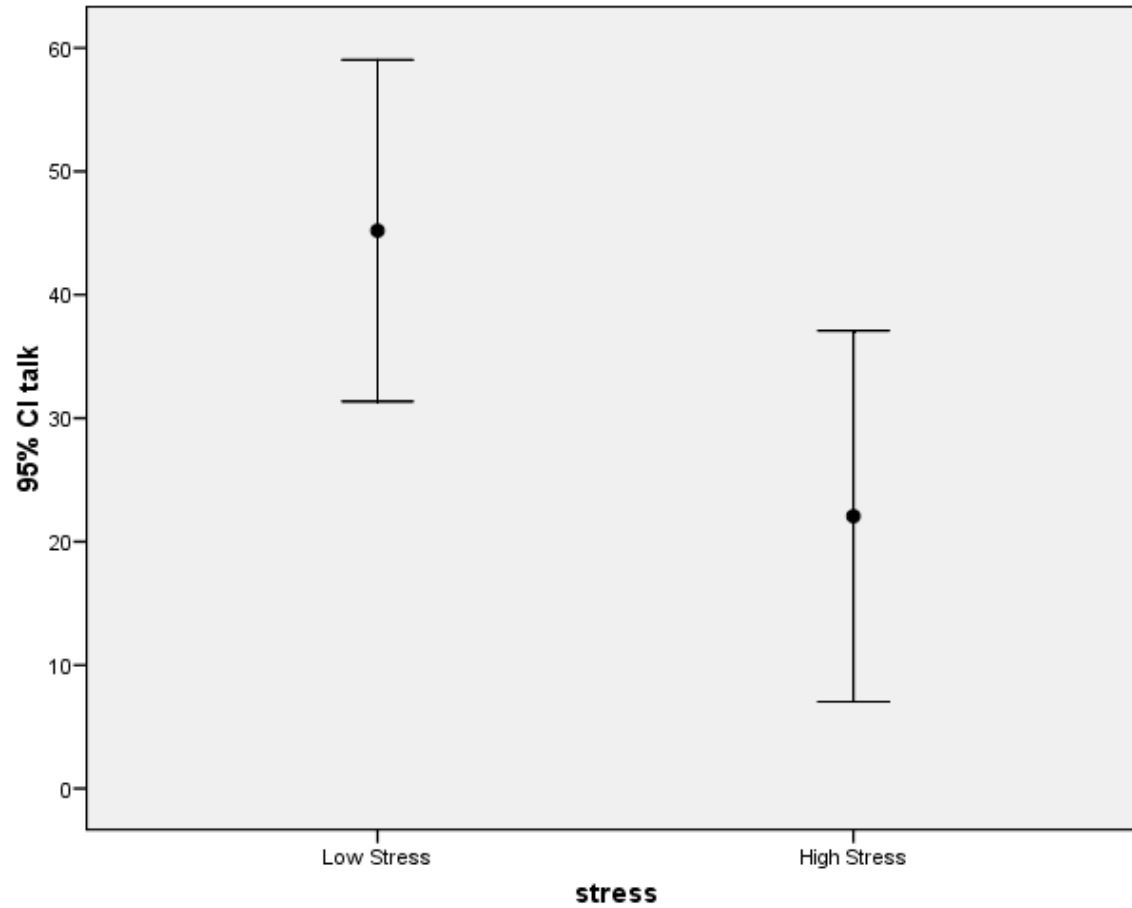
Paired Samples Test

| | | Paired Differences | | | | | | | |
|--------|-----------------|--------------------|----------------|-----------------|---|--------|-------|----|-----------------|
| | | Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference | | t | df | Sig. (2-tailed) |
| | | | | | Lower | Upper | | | |
| Pair 1 | ILS 40 - ILS 60 | 14.978 | 17.266 | 2.574 | 9.791 | 20.165 | 5.819 | 44 | .000 |

Primer 2

| Low Stress | High Stress |
|------------|-------------|
| 31 | 41 |
| 5 | 93 |
| 2 | 12 |
| 78 | 0 |
| 32 | 3 |
| 17 | 29 |
| 60 | 0 |
| 30 | 2 |
| 62 | 1 |
| 79 | 4 |
| 62 | 42 |
| 65 | 18 |
| 37 | 60 |
| 58 | 24 |
| 60 | 2 |

Ali ljudje govorimo več, ko smo nervozni?



T-test za dva neodvisna vzorca

Group Statistics

| stress | N | Mean | Std. Deviation | Std. Error Mean |
|-----------------|----|-------|----------------|-----------------|
| talk Low Stress | 15 | 45.20 | 24.969 | 6.447 |
| High Stress | 15 | 22.07 | 27.136 | 7.006 |

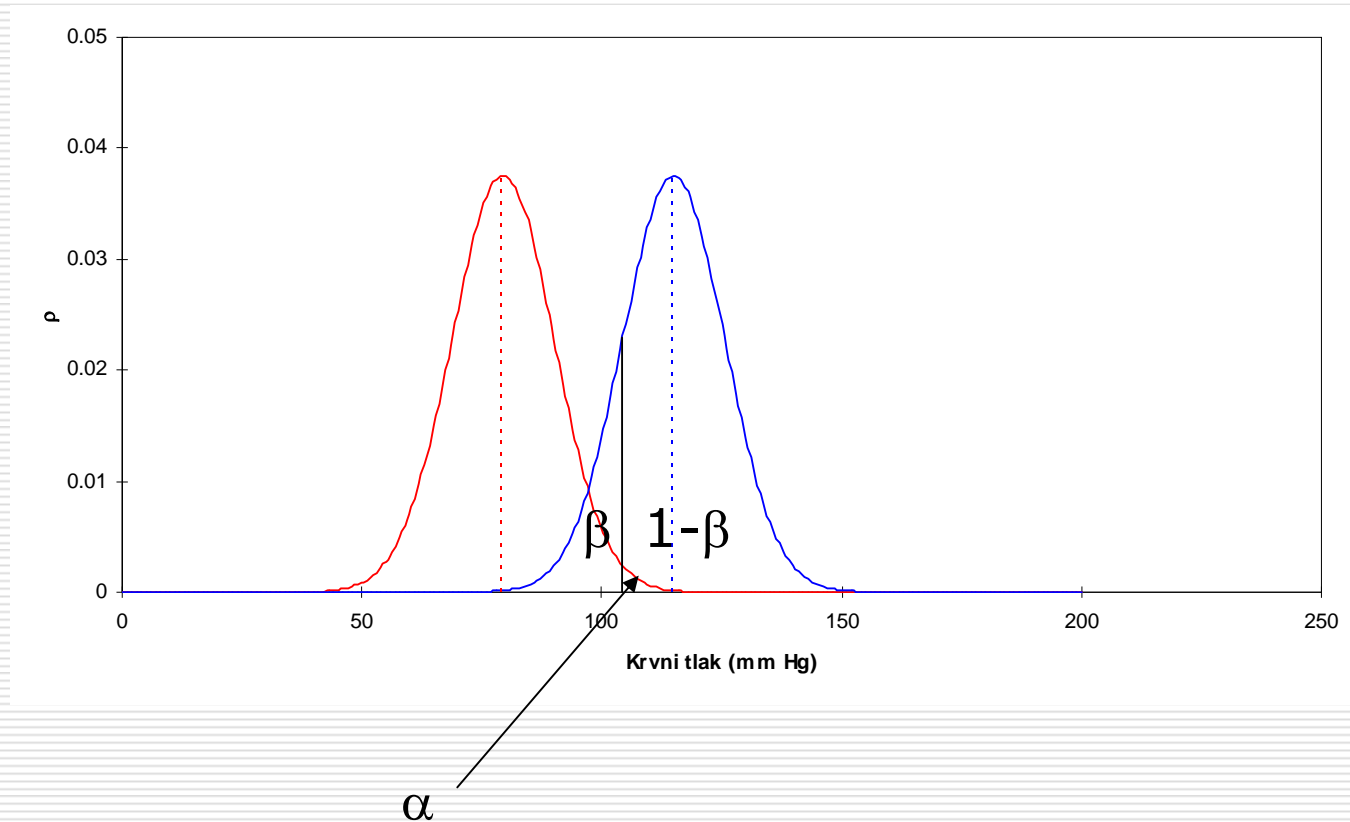
Independent Samples Test

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | | 95% Confidence Interval of the Difference | |
|------|-----------------------------|---|------|------------------------------|--------|-----------------|-----------------|-----------------------|---|--------|
| | | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower | Upper |
| talk | Equal variances assumed | .023 | .881 | 2.430 | 28 | .022 | 23.133 | 9.521 | 3.630 | 42.637 |
| | Equal variances not assumed | | | 2.430 | 27.808 | .022 | 23.133 | 9.521 | 3.624 | 42.643 |

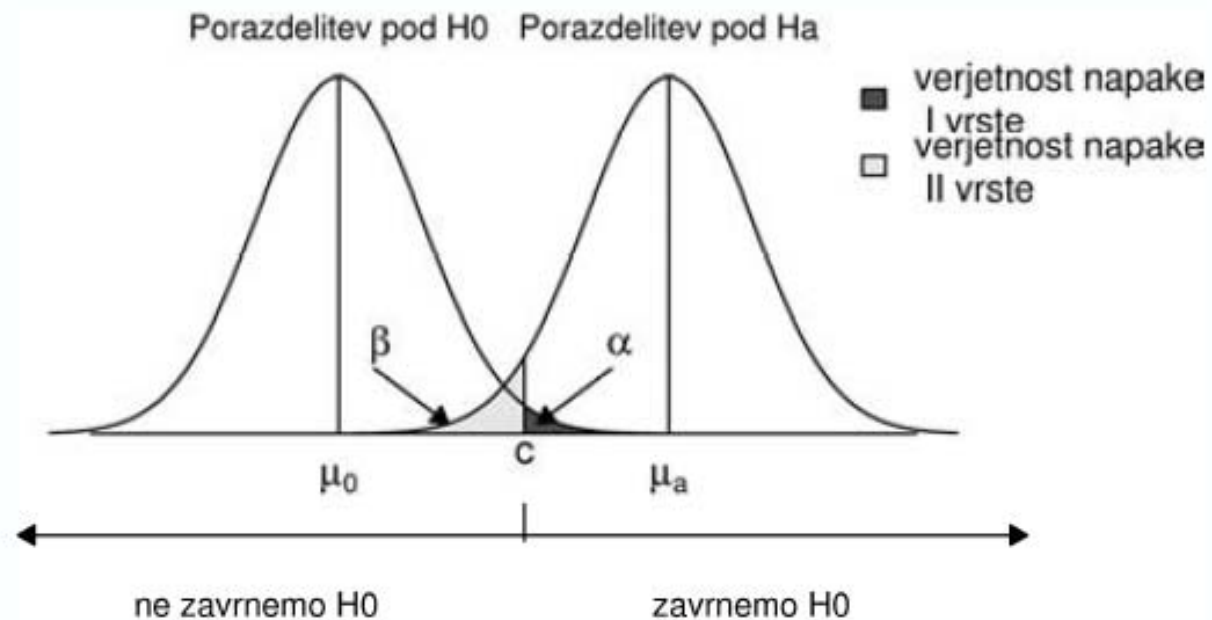
Varianci se ne razlikujeta značilno

Povprečji sta značilno različni

Moč testa

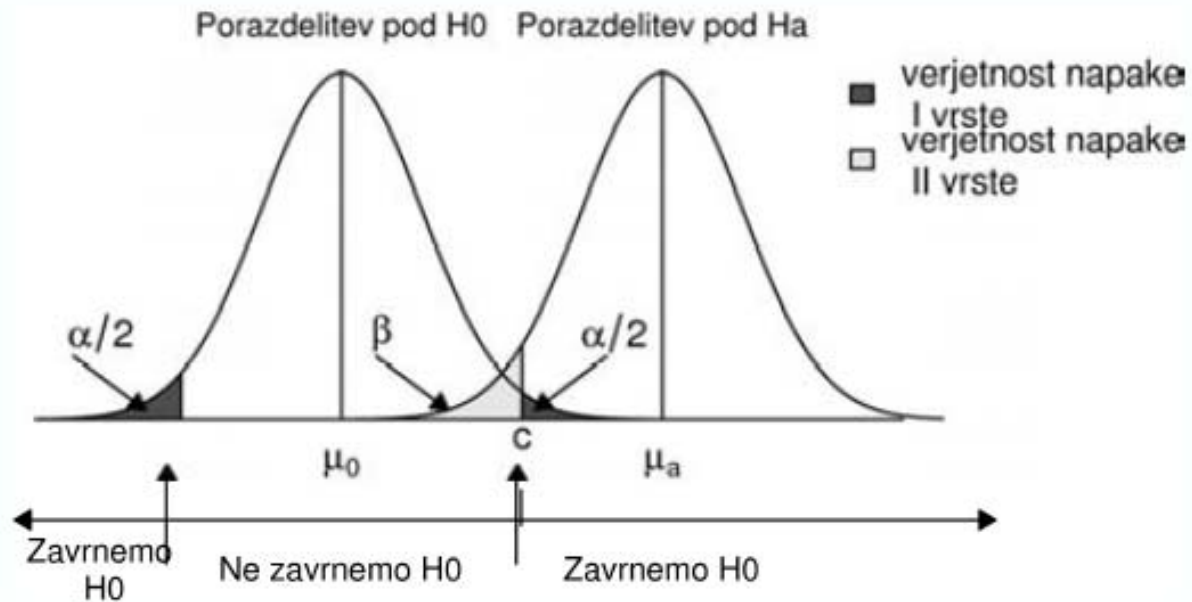


Moč testa



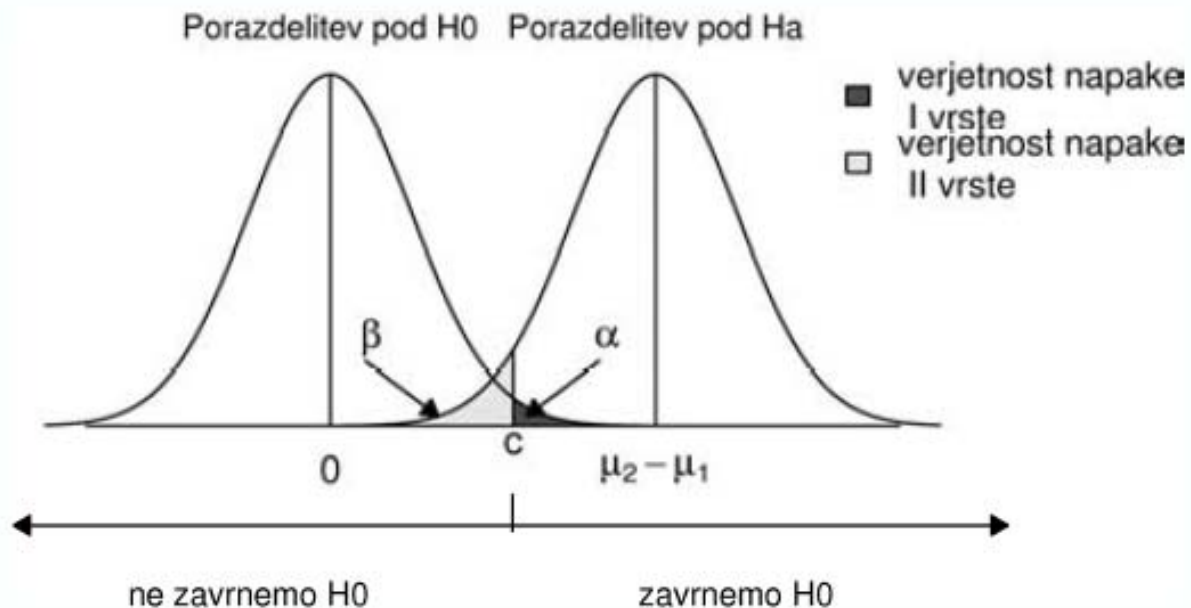
Slika 1: Testiranje hipoteze o povprečju populacije z enostranskim testom.

Dvostranski test



Slika 2: Testiranje hipoteze o povprečju populacije z dvostranskim testom.

Enostranski test



Slika 3: Dva vzorca, enostranski test za $H_0: \mu_1 = \mu_2$ proti alternativni hipotezi, da je $H_0: \mu_2 > \mu_1$.

Moč testa – poznana varianca

En vzorec

$$N = \left(\frac{\sigma}{\Delta} \right)^2 (Z_{\alpha} + Z_{\beta})^2$$

Dva neodvisna vzorca

$$N = 2 \left(\frac{\sigma}{\Delta} \right)^2 (Z_{\alpha} + Z_{\beta})^2$$

Moč testa – varianca ni poznana

En vzorec

$$N = \left(\frac{s}{\Delta}\right)^2 (Z_{\alpha} + Z_{\beta})^2 + 0.5Z_{\alpha}^2$$

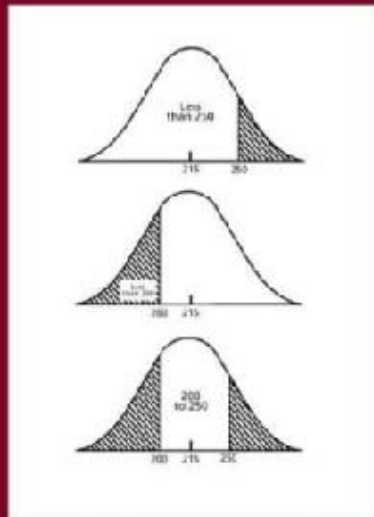
Dva neodvisna vzorca

$$N = 2\left(\frac{s}{\Delta}\right)^2 (Z_{\alpha} + Z_{\beta})^2 + 0.25Z_{\alpha}^2$$

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Practical and Clinical Applications

Fourth Edition, Revised and Expanded



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Poglavji 5 in 6!