Univerza v Ljubljani
Fakulteta za farmacijo

Centralni limitni izrek in intervalna ocena

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Marec, 2009

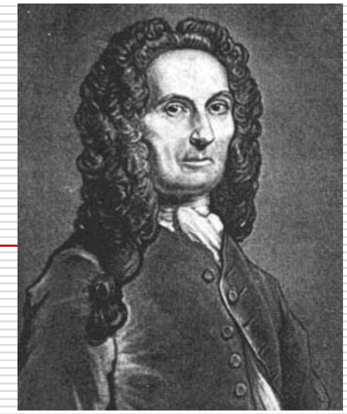
Normalna porazdelitev

- 1733 de Moivre (aproksimacija binomske porazdelitve za velike n)

- 1809 Gauss

- Pomembna zaradi ***centralnega limitnega izreka***, ki pravi, da je vsota velikega števila neodvisnih slučajnih spremenljivk (binomska porazdelitev, Poissonova porazdelitev, ...) porazdeljena normalno

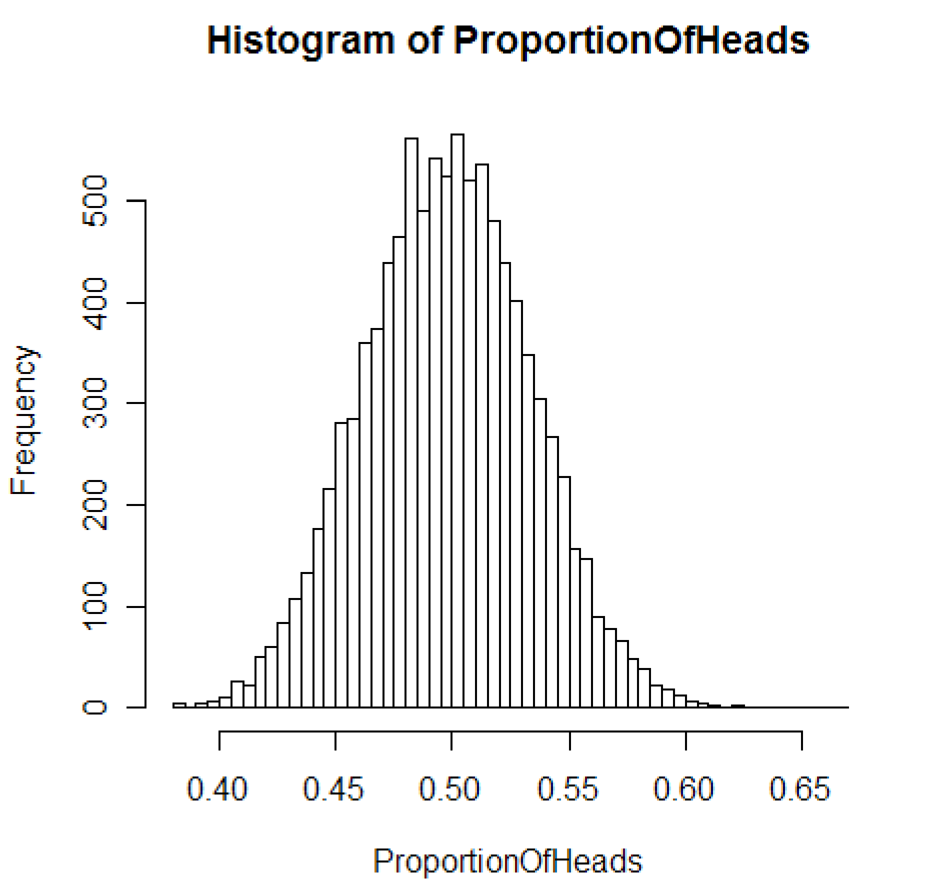
  - Primer: Telesna teža človeka je odvisna od številnih dejavnikov (genetski in okoljski, njihovi vplivi so aditivni. Telesna teža je zato porazdeljena normalo.



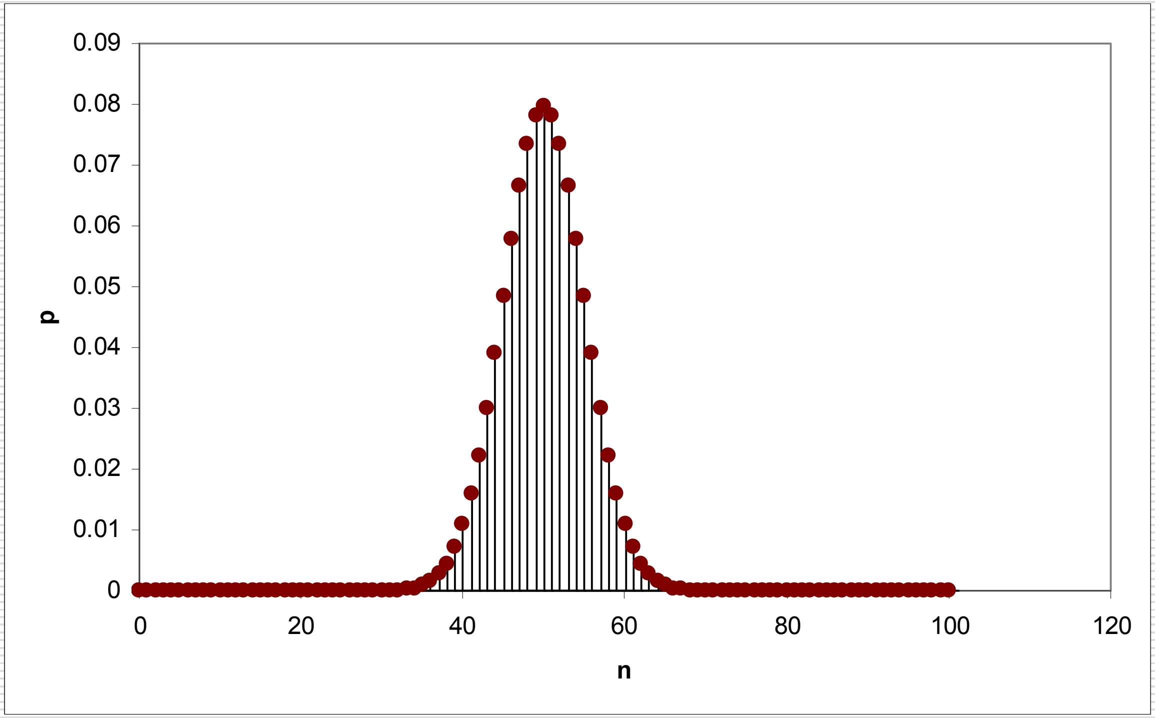
Abraham de Moivre
(1667-1754)

Karl F. Gauss
(1777-1855)

Binomska porazdelitev



Veliki vzorci



N=100
P=0.5

Povprečna lega in razpršenost

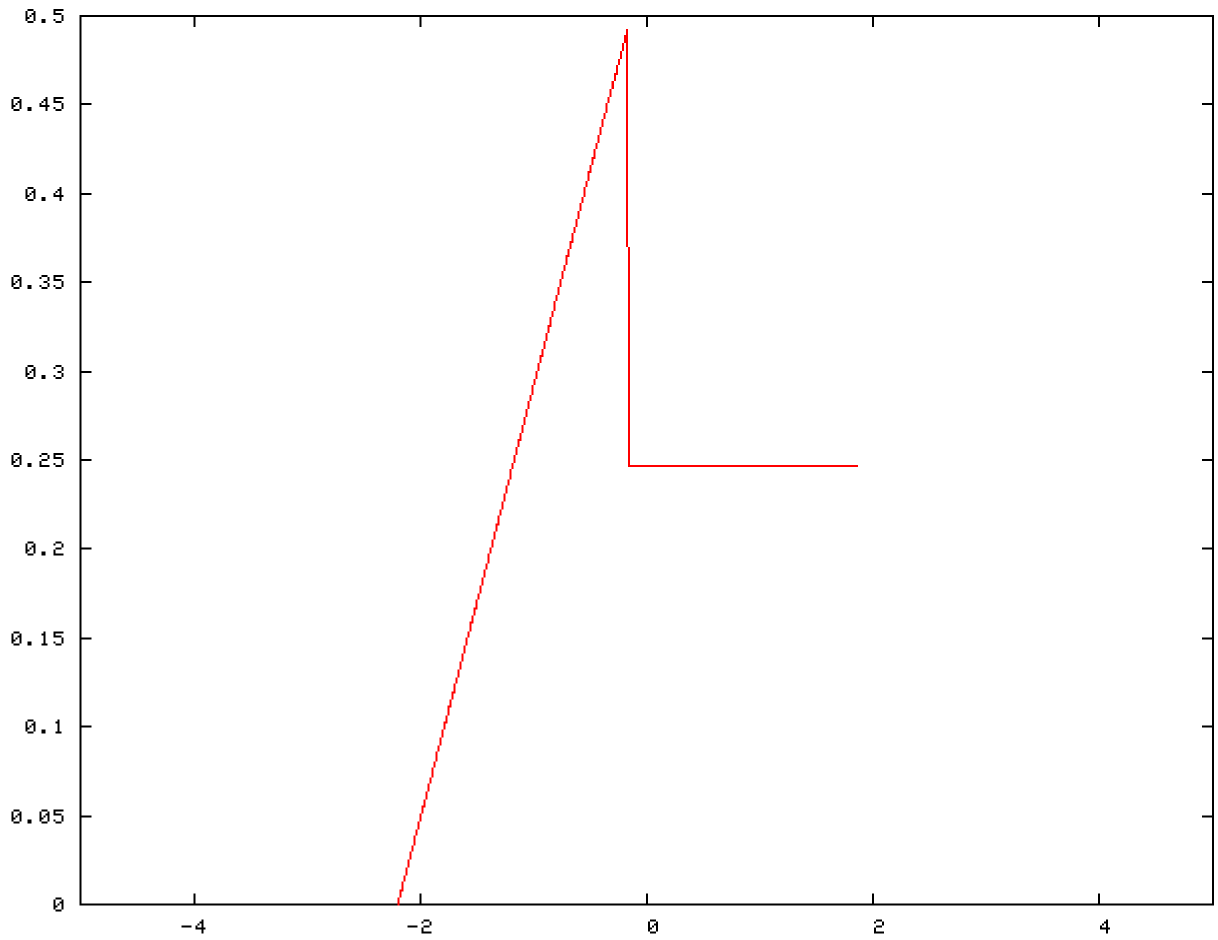
- E(x)=*n p*
- D(x)=*n p (*1-*p)*

$$\sigma = \sqrt{np(1-p)}$$

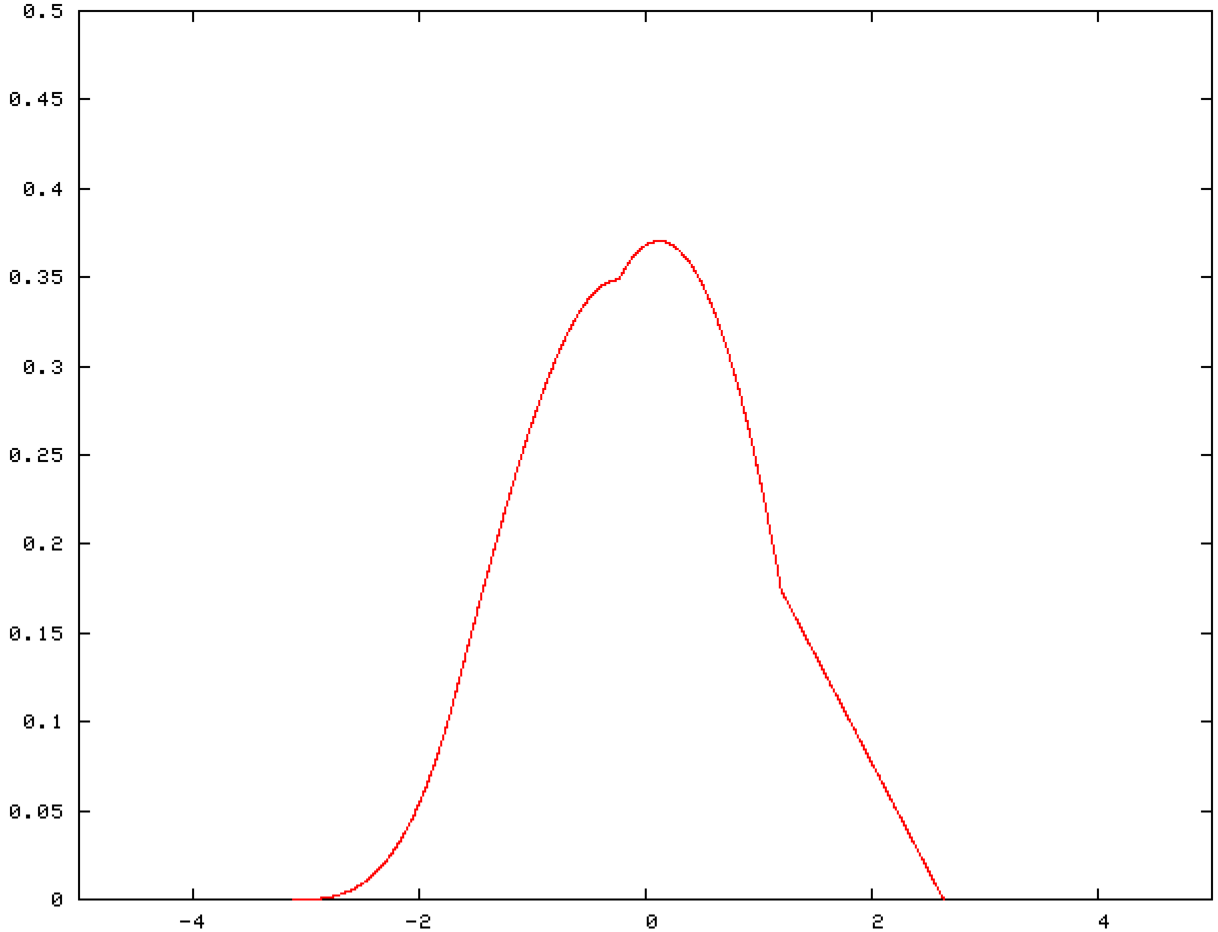
- E(x)=*p*
- D(x)=*p (*1-*p)/n*

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

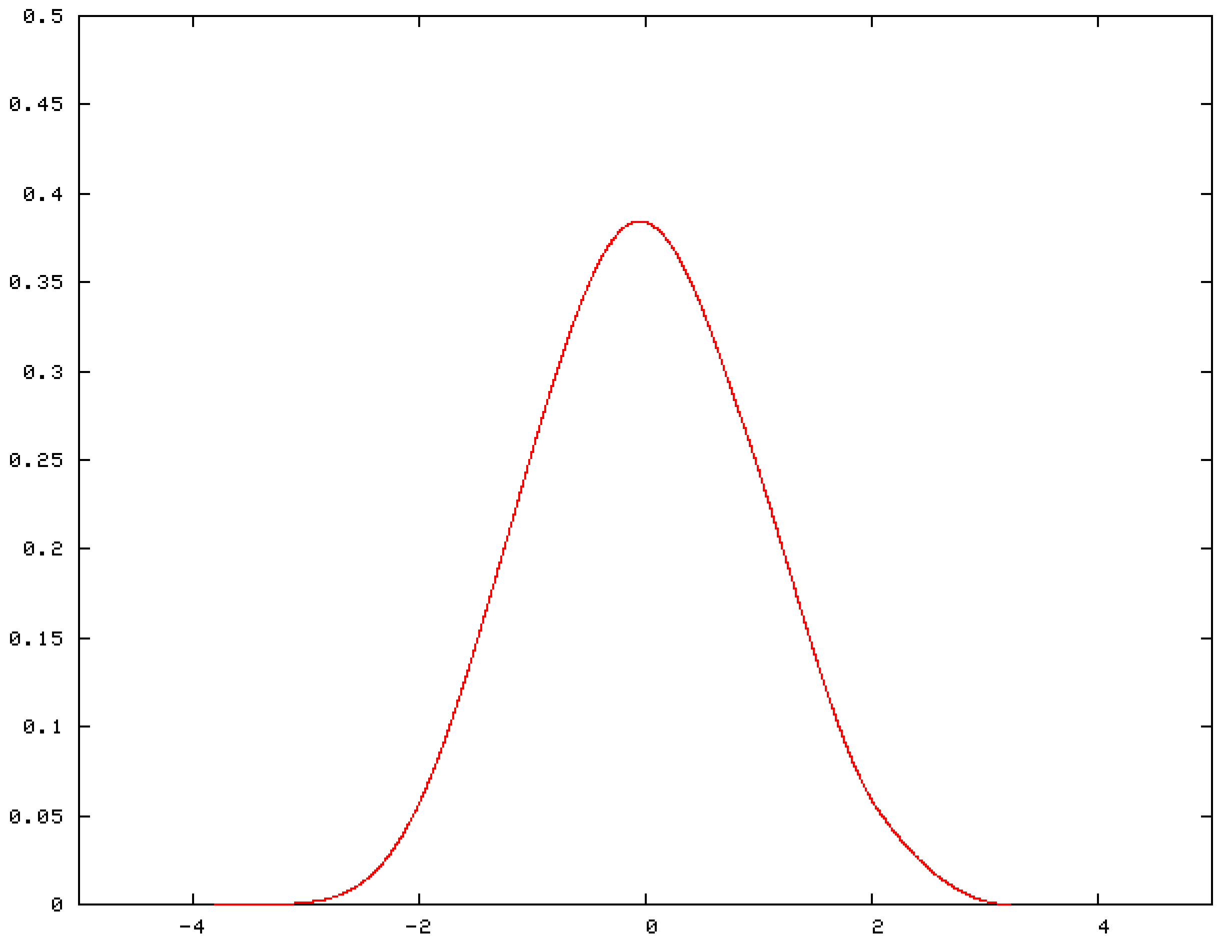
Gostota verjetnosti X



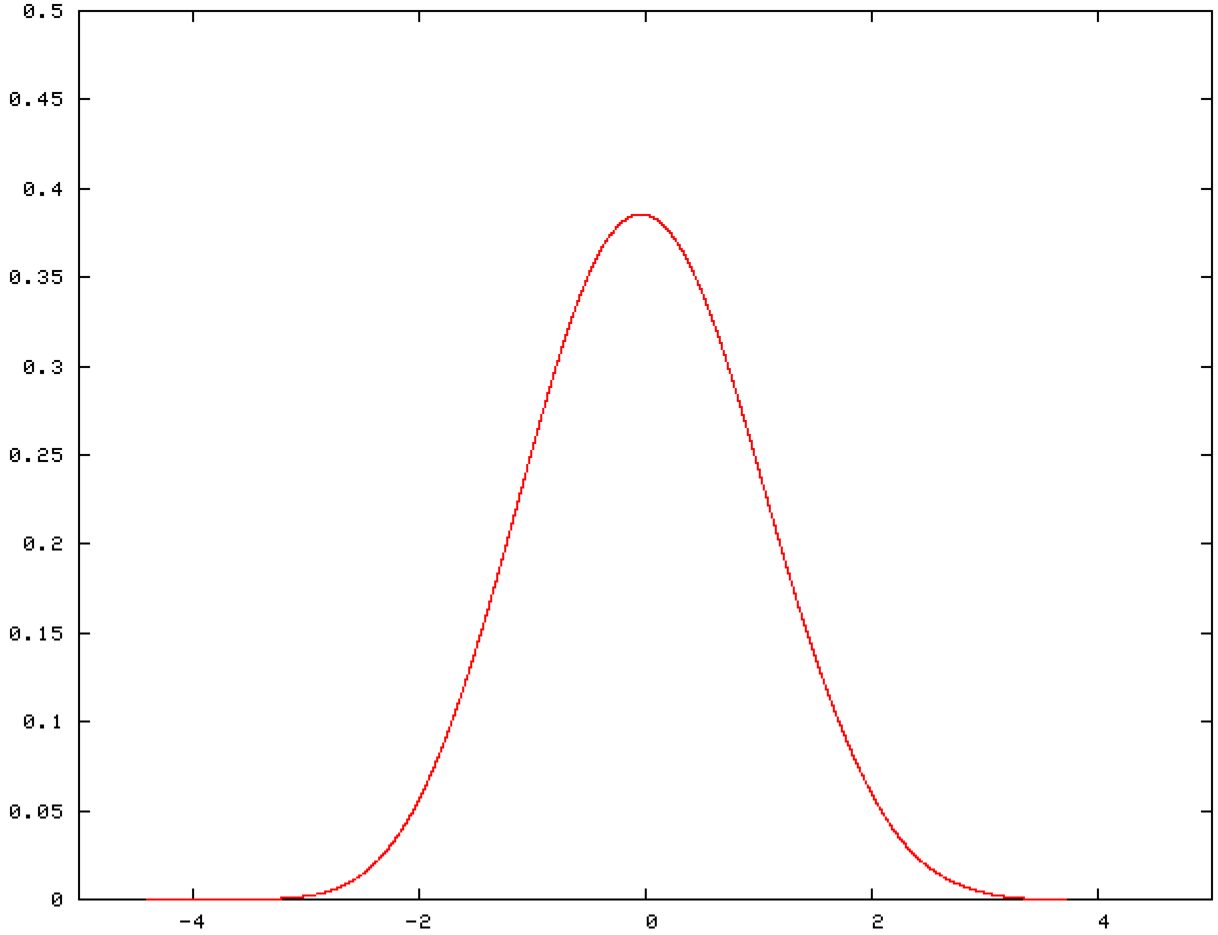
Gostota verjetnosti X_1+X_2



Gostota verjetnosti $X_1+X_2+X_3$



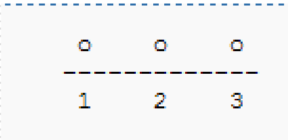
Gostota verjetnosti $X_1+X_2+X_3+X_4$



Diskretna spremenljivka

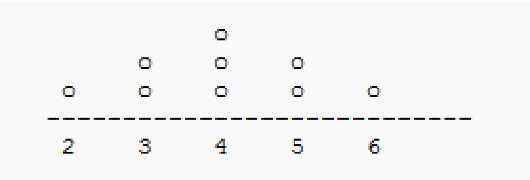
$$X = \begin{cases} 1 & \text{with probability } 1/3, \\ 2 & \text{with probability } 1/3, \\ 3 & \text{with probability } 1/3. \end{cases}$$

Enakomerna porazdelitev



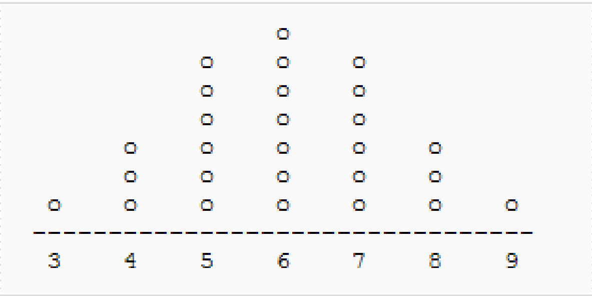
X_1+X_2

$$\left\{\begin{pmatrix} 1+1 & = & 2 \\ 1+2 & = & 3 \\ 1+3 & = & 4 \\ 2+1 & = & 3 \\ 2+2 & = & 4 \\ 2+3 & = & 5 \\ 3+1 & = & 4 \\ 3+2 & = & 5 \\ 3+3 & = & 6 \end{pmatrix}\right\} = \left\{\begin{pmatrix} 2 & \text{with probability } 1/9 \\ 3 & \text{with probability } 2/9 \\ 4 & \text{with probability } 3/9 \\ 5 & \text{with probability } 2/9 \\ 6 & \text{with probability } 1/9 \end{pmatrix}\right\}$$



$X_1 + X_2 + X_3$

$$\left\{\begin{array}{rcl}
1+1+1 & = & 3\\
1+1+2 & = & 4\\
1+1+3 & = & 5\\
1+2+1 & = & 4\\
1+2+2 & = & 5\\
1+2+3 & = & 6\\
1+3+1 & = & 5\\
1+3+2 & = & 6\\
1+3+3 & = & 7\\
2+1+1 & = & 4\\
2+1+2 & = & 5\\
2+1+3 & = & 6\\
2+2+1 & = & 5\\
2+2+2 & = & 6\\
2+2+3 & = & 7\\
2+3+1 & = & 6\\
2+3+2 & = & 7\\
2+3+3 & = & 8\\
3+1+1 & = & 5\\
3+1+2 & = & 6\\
3+1+3 & = & 7\\
3+2+1 & = & 6\\
3+2+2 & = & 7\\
3+2+3 & = & 8\\
3+3+1 & = & 7\\
3+3+2 & = & 8\\
3+3+3 & = & 9
\end{array}\right\} = \left\{\begin{array}{ll}
3 & \text{with probability } 1/27\\
4 & \text{with probability } 3/27\\
5 & \text{with probability } 6/27\\
6 & \text{with probability } 7/27\\
7 & \text{with probability } 6/27\\
8 & \text{with probability } 3/27\\
9 & \text{with probability } 1/27
\end{array}\right\}$$



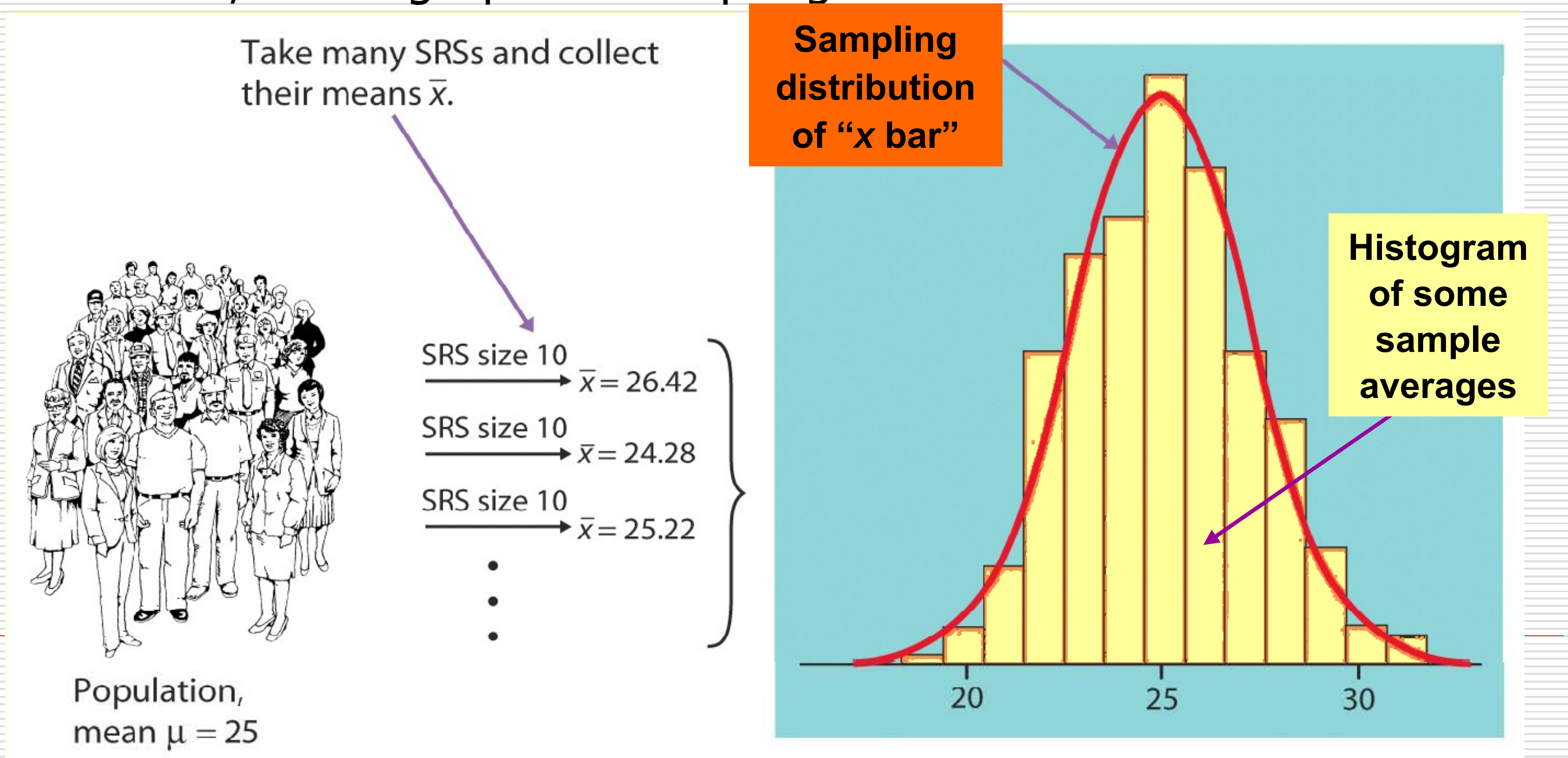
Porazdelitev vzorčnih aritmetičnih sredin
Vzorčna porazdelitev

Vzorčna porazdelitev **statistike** je njena gostota verjetnosti za neskončno veliko vzorcev velikosti N iz populacije.

Porazdelitev vzorčnih aritmetičnih sredin

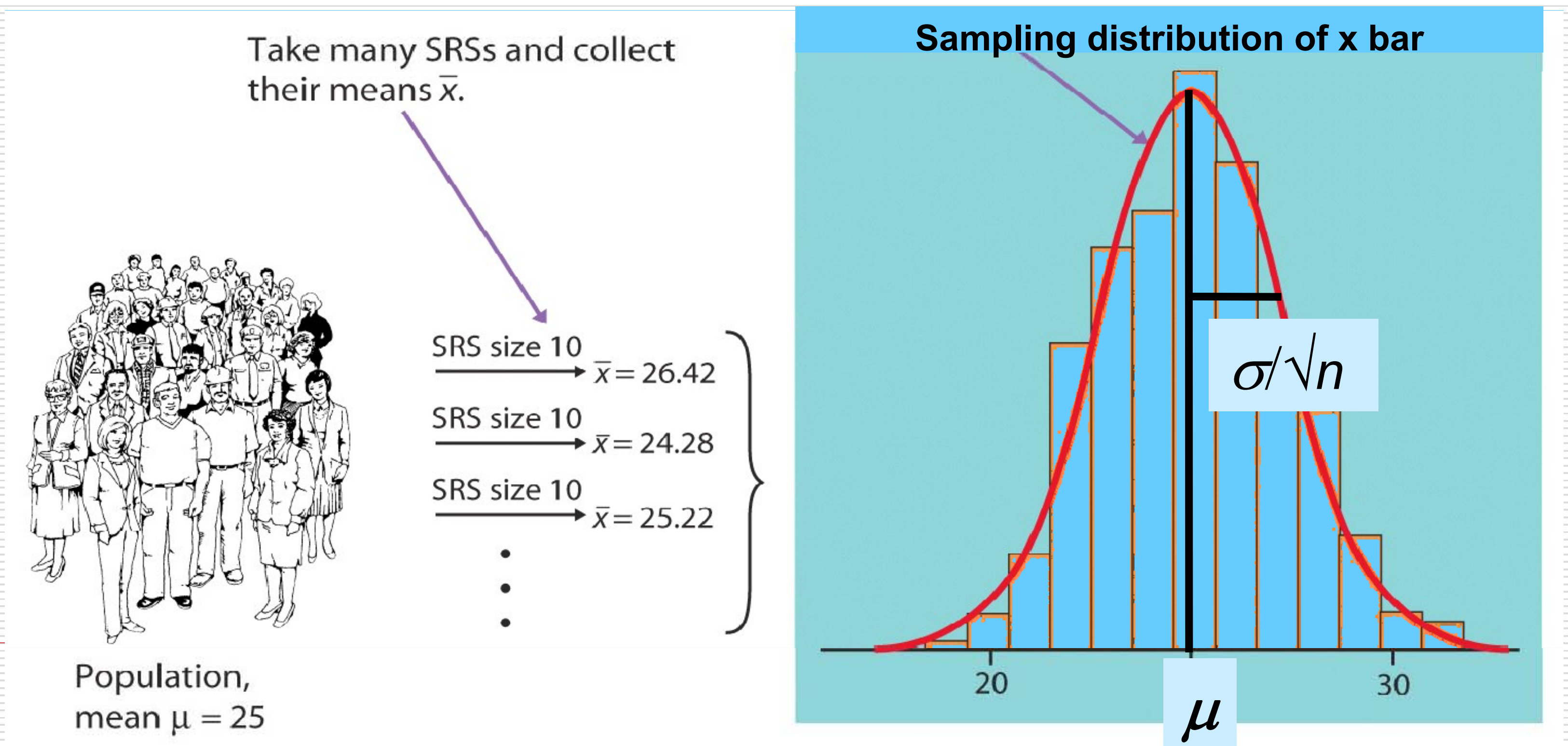
We take many random samples of a given size *n* from a population with mean μ and standard deviation σ.

Some sample means will be above the population mean μ and some will be below, making up the sampling distribution.



Za vsako populacijo z aritmetično sredino μ in standardnim odklonom σ velja

- **Aritmetična sredina** ali centralna lega vzorčne porazdelitve, je enaka aritmetični sredini populacije μ : $\mu_x = \mu$.
- **Standardni odklon** vzorčne porazdelitve pa je $\sigma/\sqrt{n}$, kjer je n velikost vzorca in σ standardni odklon v populaciji : $\sigma_x = \sigma/\sqrt{n}$.



- Aritmetična sredina vzorčne porazdelitve:

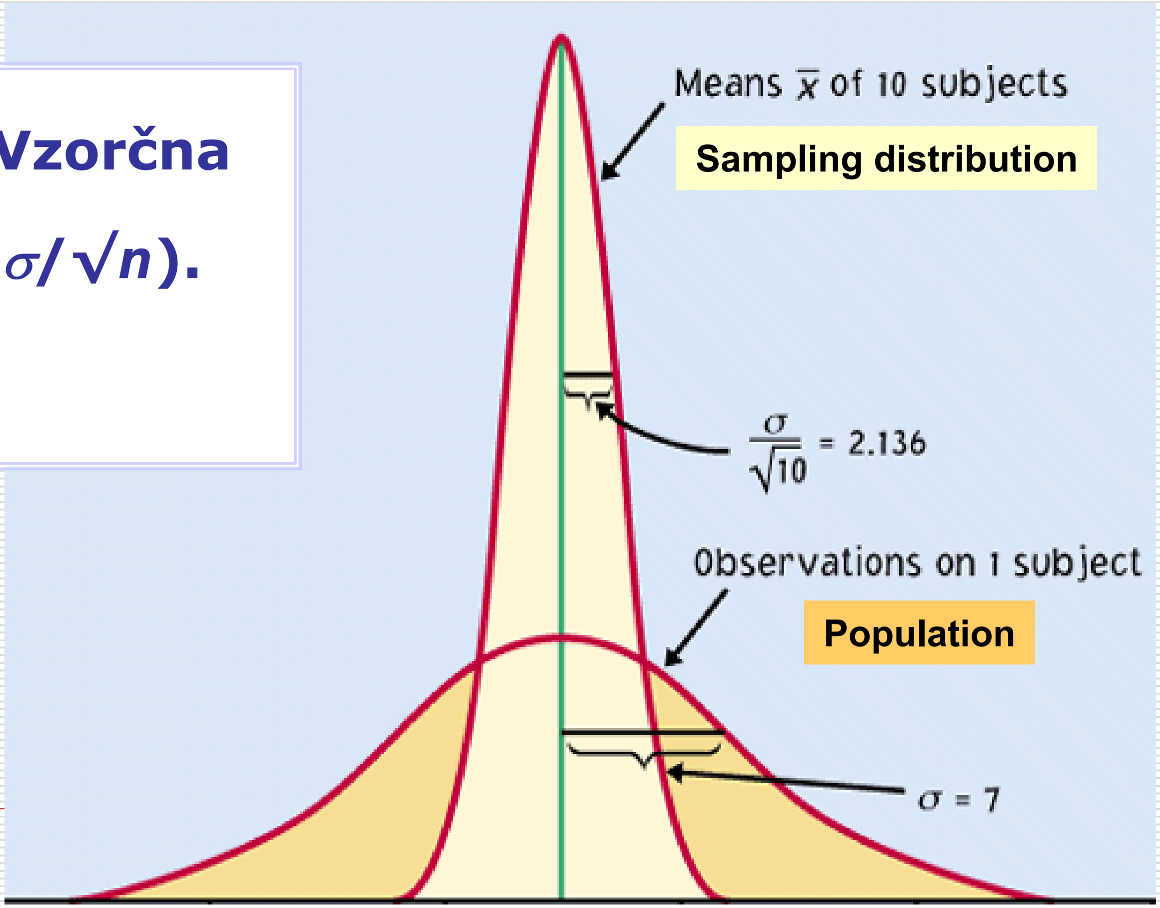
  Je nepristranska ocena za **aritmetično sredino populacije** μ.

- Standardni odklon vzorčne porazdelitve:

  Standardna napaka $\sigma / \sqrt{n}$. ➔ **Variabilnost povprečij je manjša od variabilnosti posameznih opazovanj**.

Normalno porazdeljena spremenljivka

Populacija $N(\mu, \sigma)$ Vzorčna porazdelitev $N(\mu, \sigma/\sqrt{n})$.



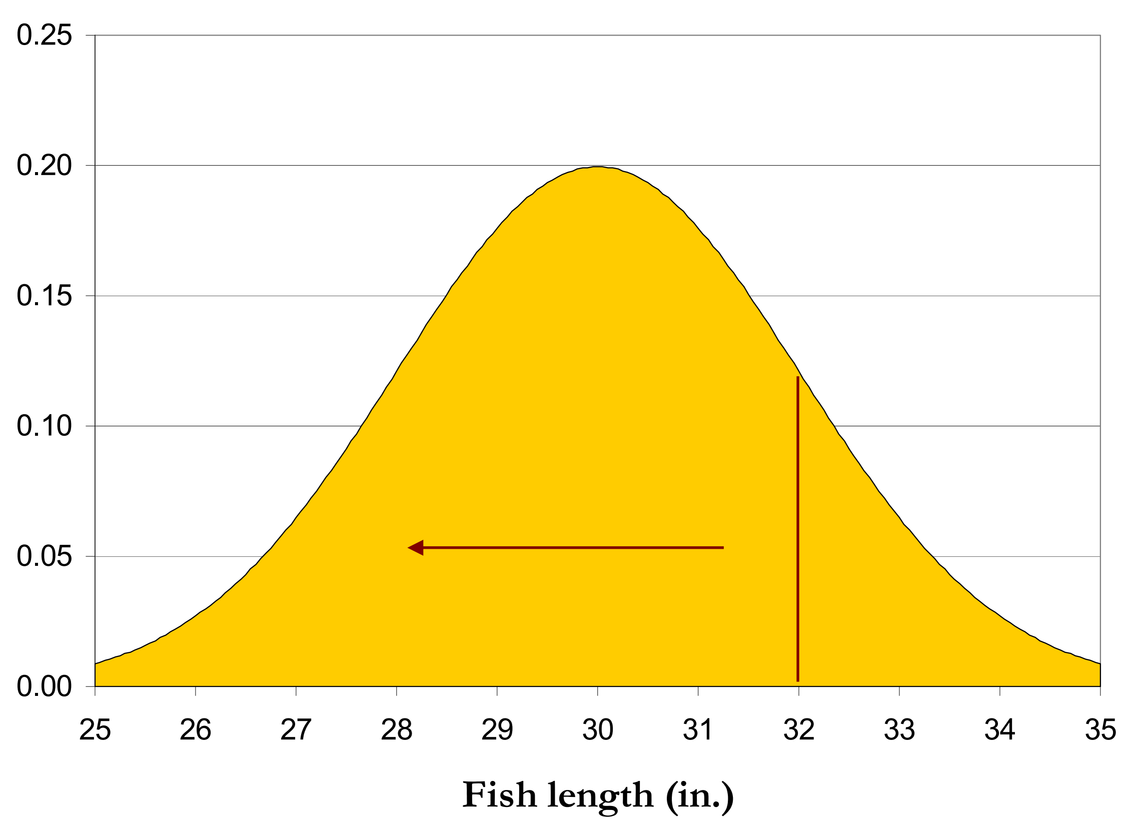
EXAMPLE

A certain brand of tires has a mean life of 25,000 miles with a standard deviation of 1600 miles.

What is the probability that the mean life of 64 tires is less than 24,600 miles?

Normalna porazdelitev

- Kakšna je verjetnost, da bo ujeta postrv krajša od 32 cm?



Example continued

The sampling distribution of the means has a mean of 25,000 miles (the population mean)

μ = 25000 mi.

and a standard deviation (i.e.. standard error) of:

1600/8 = 200

Example continued

Convert 24,600 mi. to a z-score and use the normal table to determine the required probability.

$$z = (24600-25000)/200 = -2$$

$$P(z < -2) = 0.0228$$

or 2.28% of the sample means will be less than 24,600 mi.

ESTIMATION OF POPULATION VALUES

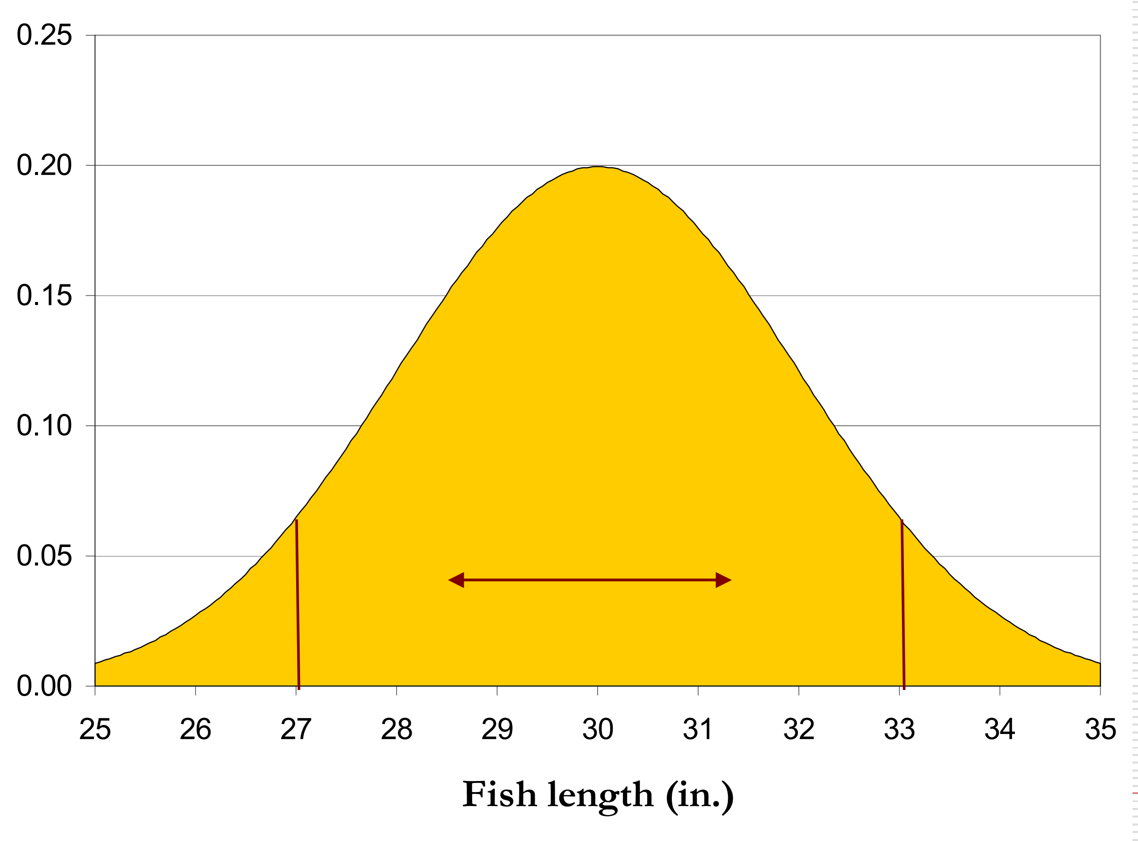
- Point Estimates
- Interval Estimates

CONFIDENCE INTERVAL ESTIMATES for LARGE SAMPLES

- The sample has been randomly selected
- The population standard deviation is known or the sample size is at least 30.

Normalna porazdelitev

- Kakšna je verjetnost, da bo ujeta postrv dolga med 26 in 29 cm?



Interval zaupanja za aritmetično sredino populacije

$$\bar{X} - z\frac{s}{\sqrt{n}} \le \mu \le \bar{X} + z\frac{s}{\sqrt{n}}$$

$\bar{X}$: sample mean

s: sample standard deviation

n: sample size

EXAMPLE

Estimate, with 95% confidence, the lifetime of nine volt batteries using a randomly selected sample where:

$\bar{X}$ = 49 hours

s = 4 hours

n = 36

EXAMPLE continued

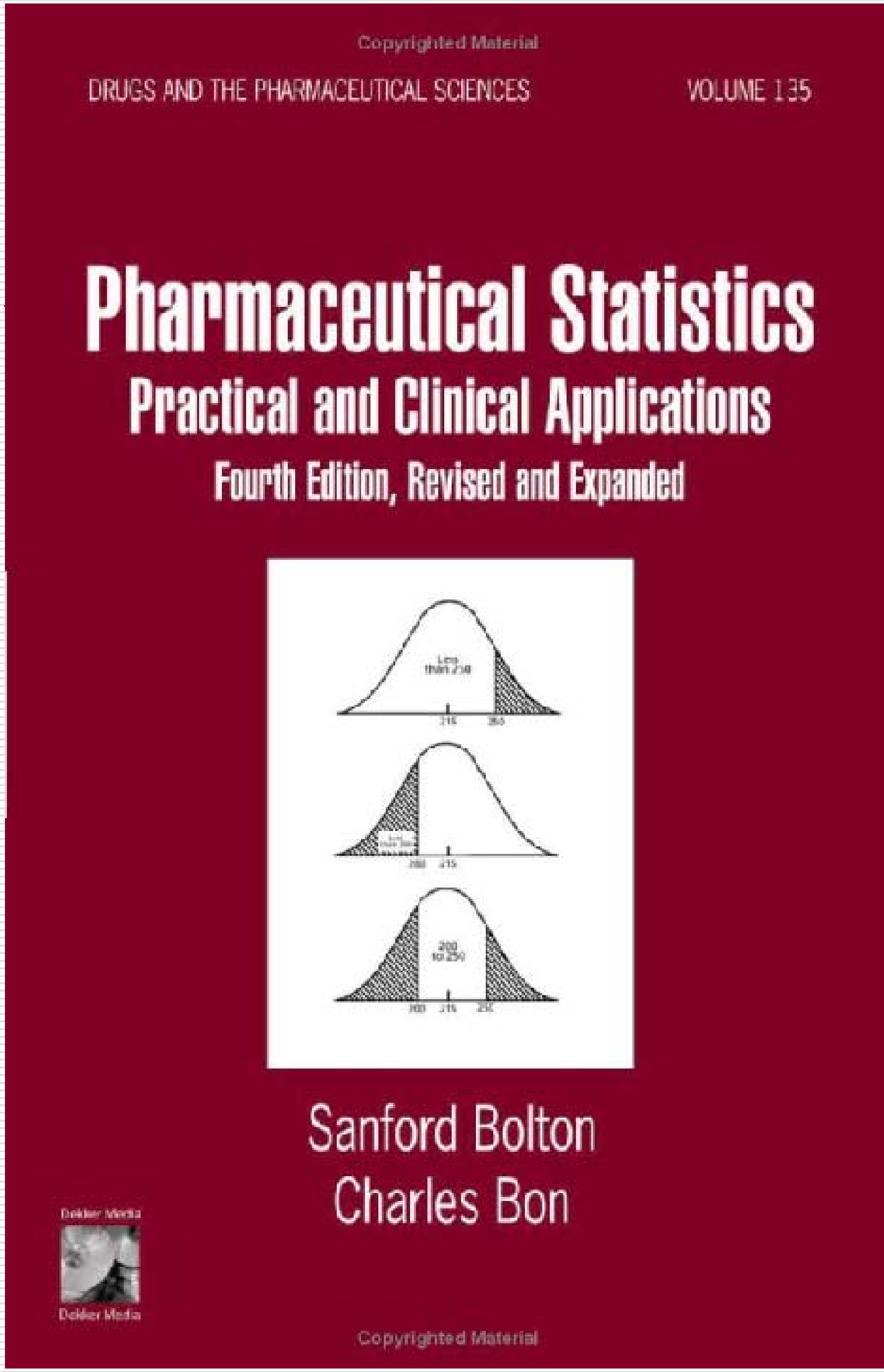
Lower Limit: $49 - (1.96)(4/6)$

$49 - (1.3) = 47.7$ hrs

Upper Limit: $49 + (1.96)(4/6)$

$49 + (1.3) = 50.3$ hrs

We are 95% confident that the mean lifetime of the population of batteries is between 47.7 and 50.3 hours.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